FYJC - MATHEMATICS & STATISTICS

PAPER - I

Compiled & Conducted @ JKSC

CONICS - CIRCLE

- ✓ Standard form of the circle : $x^2 + y^2 = r^2$

- ✓ Given centre $C \equiv (h,k)$ & radius = r , equation of the circle can be generated using

  Center Radius form

  $(x - h)^2 + (y - k)^2 = r^2$

- ✓ Given $A(x_1,y_2)$, $B(x_2,y_2)$ are the ends of the diameter , equation of the circle can be generated using

  Diameter Form

  $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

- ✓ In general , equation of the circle

  $x^2 + y^2 + 2gx + 2fy + c = 0$, where

  $C \equiv (-g,-f)$ $R = \sqrt{g^2 + f^2 - c}$

Q1.

01. find the equation of the circle with center (2 , –3) and passing through (–1 , 2)

    ans : $x^2 + y^2 - 4x + 6y - 21 = 0$

02. find the equation of the circle with center (1 , –2) and passing through (5 , 3)

    ans : $x^2 + y^2 - 2x + 4y - 36 = 0$

03. find the equation of the circle with center (–2 , 3) and passing through (1 , 7)

    ans : $x^2 + y^2 + 4x - 6y - 12 = 0$

04. find the equation of the circle with center (1⁄2 , 3⁄2) and radius 3

    ans : $2x^2 + 2y^2 - 2x - 6y - 13 = 0$

Q2.

01. find equation of circle with radius 5 and concentric with circle $x^2 + y^2 + 4x - 6y = 0$

    ans : $x^2 + y^2 + 4x - 6y - 12 = 0$

02. find equation of circle with radius 5 and concentric with circle $x^2 + y^2 - 6x - 4y - 3 = 0$

    ans : $x^2 + y^2 - 6x - 4y - 12 = 0$

03. find equ. of circle concentric with $x^2 + y^2 - 2x - 6y - 7 = 0$ and area 616 sq. units

    ans : $x^2 + y^2 - 2x - 6y - 186 = 0$

04. find equ. of circle concentric with $x^2 + y^2 - 6x + 60 = 0$ and circumference 4π

    ans : $x^2 + y^2 - 6x + 5 = 0$ **(MAR 2016)**

05. Find centre and radius of the circle : $2x^2 + 2y^2 - 2x - 8y - 13 = 0$ **(MAR 2014)**

ans : $C(^1/_2 , 2)$, $r = \sqrt{43} / 2$

06. Find centre and radius of the circle : $3x^2 + 3y^2 - 6x + 4y - 42 = 0$

ans : $C(1 , ^{-2}/_3)$, $r = \sqrt{22} / 3$

07. find the center and the radius of the circle : $(x - 3)(x - 5) + (y - 1)(y - 7) = 0$

ans : $C(4 , 4)$, $r = \sqrt{10}$

Q3.

01. find equation of the circle having centre (7,−2) and touching the x − axis

ans : $x^2 + y^2 - 14x + 4y + 49 = 0$

02. find equation of the circle having centre (−5,2) and touching the y − axis

ans : $x^2 + y^2 + 10x - 4y + 4 = 0$

03. find equation of the circle having radius = 1 and touching the x − axis at (−4,0)

ans : $x^2 + y^2 + 8x \pm 2y + 16 = 0$

Q4.

01. Find circle touching both the axes and having radius 7

ans : $x^2 + y^2 \pm 14x \pm 14y + 49 = 0$

02. find equation of the circle touching both axes and passing through (1,2)

ans : $x^2 + y^2 - 2x - 2y + 1 = 0$, $x^2 + y^2 - 10x - 10y + 25 = 0$

03. find equation of the circle touching both axes and passing through (−9,8)

ans : $x^2 + y^2 + 10x - 10y + 25 = 0$, $x^2 + y^2 + 58x - 58y + 841 = 0$

Q5.

01. Find equation of circle with center (4,3) & touching $5x - 12y - 10 = 0$

ans : $x^2 + y^2 - 8x - 6y + 21 = 0$

02. Find equation of circle with center (3,1) & touching $8x - 15y + 25 = 0$

ans : $x^2 + y^2 - 6x - 2y + 6 = 0$

Q6.

01. Find equation of circle passing through (4 , 6) ; (–3 , 5) & (5 , –1)

ans : $x^2 + y^2 - 2x - 4y - 20 = 0$

02. Find equation of circle passing through (4 , 1) ; (–3 , –6) & (–2 , 1) **(MAR 2013)**

ans : $x^2 + y^2 - 2x + 6y - 15 = 0$

03. Find equation of circle passing through (4 , 1) ; (6 , 5) & whose center lies on $4x + y = 16$

ans : $x^2 + y^2 - 6x - 8y + 15 = 0$

04. Find equation of circle passing through (1 , –4) ; (5 , 2) & whose center lies on $x - 2y + 9 = 0$

ans : $x^2 + y^2 + 6x - 6y - 47 = 0$

Q7.

01. find equation of circle passing through (1 , 9) & touching $3x + 4y + 6 = 0$ at (–2 , 0)

ans : $x^2 + y^2 - 2x - 8y - 8 = 0$

02. find equation of circle passing through (–1 , –3) & touching $4x + 3y - 12 = 0$ at (3 , 0)

ans : $x^2 + y^2 - 2x + 3y - 3 = 0$

Q8.

01. Find equation of circle with center (3 , –1) and which cuts off a chord of length 6 on line $2x - 5y + 18 = 0$

ans : $x^2 + y^2 - 6x + 2y - 28 = 0$

02. Find equation of circle with center (1 , 4) and which cuts off a chord of length 6 on line $3x + 4y + 1 = 0$

ans : $x^2 + y^2 - 2x - 8y - 8 = 0$

03. Find the length of intercept made by circle $x^2 + y^2 - 2x - 8y - 8 = 0$ on the line $3x + 4y + 1 = 0$

ans : 6

04. Find the length of intercept made by circle $x^2 + y^2 - 6x + 4y - 12 = 0$ on the line $4x - 3y + 2 = 0$

ans : 6

MARCH – 2015

line $2x - y + 6 = 0$ meets the circle $x^2 + y^2 - 2x - 12 = 0$ at A and B . Find the equation of circle on AB as diameter

MARCH – 2017

Find equation of circle passing through point of intersection of the lines $x + 3y = 0$ and $2x - 7y = 0$ and whose centre is the point of intersection of the lines $x + y + 1 = 0$ and $x - 2y + 4 = 0$

SOLUTION SET

Q1.

01. find the equation of the circle with center (2 , –3) and passing through (–1 , 2)

SOLUTION

STEP 1 :

$r = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(2 + 1)^2 + (-3 - 2)^2}$

$= \sqrt{9 + 25}$

$= \sqrt{34}$

STEP 2 :

$C(2,-3)$, $r = \sqrt{34}$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 2)^2 + (y + 3)^2 = (\sqrt{34})^2$

$x^2 - 4x + 4 + y^2 + 6y + 9 = 34$

$x^2 + y^2 - 4x + 6y + 13 - 34 = 0$

$x^2 + y^2 - 4x + 6y - 21 = 0$ …….. equation of the circle

02. find the equation of the circle with center (1 , –2) and passing through (5 , 3)

SOLUTION

STEP 1 :

$r = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(1 - 5)^2 + (-2 - 3)^2}$

$= \sqrt{16 + 25}$

$= \sqrt{41}$

STEP 2 :

$C(1,-2)$, $r = \sqrt{41}$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 1)^2 + (y + 2)^2 = (\sqrt{41})^2$

$x^2 - 2x + 1 + y^2 + 4y + 4 = 41$

$x^2 + y^2 - 2x + 4y + 5 - 41 = 0$

$x^2 + y^2 - 2x + 4y - 36 = 0$ …….. equation of the circle

03. find the equation of the circle with center (–2 , 3) and passing through (1 , 7)

SOLUTION

STEP 1 :

$$r = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(-2 - 1)^2 + (3 - 7)^2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25} = 5$$

STEP 2 :

$C(-2,3)$, $r = 5$

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x + 2)^2 + (y - 3)^2 = (5)^2$$

$$x^2 + 4x + 4 + y^2 - 6y + 9 = 25$$

$$x^2 + y^2 + 4x - 6y + 13 - 25 = 0$$

$x^2 + y^2 + 4x - 6y - 12 = 0$ …….. equation of the circle

04. find the equation of the circle with center (1⁄2 , 3⁄2) and radius 3

SOLUTION

STEP 1 :

$C(1/2 , 3/2)$, $r = 3$

$$(x - h)^2 + (y - k)^2 = r^2$$

$$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 = 3^2$$

$$\left(\frac{2x - 1}{2}\right)^2 + \left(\frac{2y - 3}{2}\right)^2 = 9$$

$$\frac{4x^2 - 4x + 1}{4} + \frac{4y^2 - 12y + 9}{4} = 9$$

$$4x^2 + 4y^2 - 4x - 12y + 10 = 36$$

$$4x^2 + 4y^2 - 4x - 12y - 26 = 0$$

$2x^2 + 2y^2 - 2x - 6y - 13 = 0$ …….. equation of the circle

Q2.

01. find equation of circle with radius 5 and concentric with circle $x^2 + y^2 + 4x - 6y = 0$

SOLUTION

STEP 1 : $x^2 + y^2 + 4x - 6y = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = 4$; $2f = -6$

$g = 2$; $f = -3$; $c = 0$

$C \equiv (-g , -f)$

$\equiv (-2 , 3)$

STEP 2 : $C(-2,3)$, $r = 5$

$(x - h)^2 + (y - k)^2 = r^2$

$(x + 2)^2 + (y - 3)^2 = (5)^2$

$x^2 + 4x + 4 + y^2 - 6y + 9 = 25$

$x^2 + y^2 + 4x - 6y + 13 - 25 = 0$

$x^2 + y^2 + 4x - 6y - 12 = 0$ …….. equation of the circle

02. find equation of circle with radius 5 and concentric with circle $x^2 + y^2 - 6x - 4y - 3 = 0$

SOLUTION

STEP 1 : $x^2 + y^2 - 6x - 4y - 3 = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -6$; $2f = -4$

$g = -3$; $f = -2$; $c = 0$

$C \equiv (-g , -f)$

$\equiv (3,2)$

STEP 2 : $C(3,2)$, $r = 5$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 3)^2 + (y - 2)^2 = (5)^2$

$x^2 - 6x + 9 + y^2 - 4y + 4 = 25$

$x^2 + y^2 - 6x - 4y + 13 - 25 = 0$

$x^2 + y^2 - 6x - 4y - 12 = 0$ …….. equation of the circle

03. find equ. of circle concentric with $x^2 + y^2 - 2x - 6y - 7 = 0$ and area 616 sq. units

SOLUTION

STEP 1 :

$x^2 + y^2 - 2x - 6y - 7 = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -2$; $2f = -6$

$g = -1$; $f = -3$; $c = 0$

$C \equiv (-g, -f) \equiv (1,3)$

STEP 2 :

area = 616

$\pi r^2 = 616$

$r^2 = \dfrac{616}{\pi}$

$r^2 = \dfrac{616 \times 7}{22} = 196$

$r = 14$

STEP 3 :

$C(1,3)$, $r = 14$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 1)^2 + (y - 3)^2 = (14)^2$

$x^2 - 2x + 1 + y^2 - 6y + 9 = 196$

$x^2 + y^2 - 2x - 6y + 10 - 196 = 0$

$x^2 + y^2 - 2x - 6y - 186 = 0$ …….. equation of the circle

04. find equ. of circle concentric with $x^2 + y^2 - 6x + 60 = 0$ and circumference is 4π

SOLUTION

STEP 1

$x^2 + y^2 - 6x + 60 = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -6$; $2f = 0$; $c = {}^{-11}/_{3}$

$g = -3$; $f = 0$; $c = {}^{-11}/_{3}$

$C \equiv (-g, -f) \equiv (3, 0)$

STEP 2

circumference $= 4\pi$

$2\pi r = 4\pi$

$r = 2$

STEP 3 : $C(3,0)$, $r = 2$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 3)^2 + (y - 0)^2 = 2^2$

$x^2 - 6x + 9 + y^2 = 4$

$x^2 + y^2 - 6x + 5 = 0$

05. Find centre and radius of the circle : $2x^2 + 2y^2 - 2x - 8y - 13 = 0$

SOLUTION

$$2x^2 + 2y^2 - 2x - 8y - 13 = 0$$

$$x^2 + y^2 - x - 4y - \frac{13}{2} = 0$$

On comparing with

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$2g = -1 \;;\quad 2f = -4 \;;\quad c = -13/2$$

$$g = \frac{-1}{2} \;;\quad f = -2 \;;\quad c = -13/2$$

$$C \equiv (-g, -f) \qquad R = \sqrt{g^2 + f^2 - c}$$

$$\equiv \left(\frac{1}{2}, 2\right) \qquad = \sqrt{\frac{1}{4} + 4 + \frac{13}{2}}$$

$$= \sqrt{\frac{1 + 16 + 26}{4}} = \frac{\sqrt{43}}{2}$$

06. Find centre and radius of the circle : $3x^2 + 3y^2 - 6x + 4y - 3 = 0$

SOLUTION

$$3x^2 + 3y^2 - 6x + 4y - 3 = 0$$

$$x^2 + y^2 - 2x + \frac{4y}{3} - 1 = 0$$

On comparing with

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$2g = -2 \;;\quad 2f = \frac{4}{3} \;;\quad c = -1$$

$$g = -1 \;;\quad f = \frac{-2}{3} \;;\quad c = -1$$

$$C \equiv (-g, -f) \qquad R = \sqrt{g^2 + f^2 - c}$$

$$\equiv \left(1, -\frac{2}{3}\right) \qquad = \sqrt{1 + \frac{4}{9} + 1}$$

$$= \sqrt{\frac{9 + 4 + 9}{4}}$$

$$= \frac{\sqrt{22}}{3}$$

07. find the center and the radius of the circle : $(x - 3)(x - 5) + (y - 1)(y - 7) = 0$

$$(x - 3)(x - 5) + (y - 1)(y - 7) = 0$$

$$x^2 - 8x + 15 + y^2 - 8y + 7 = 0$$

$$x^2 + y^2 - 8x - 8y + 22 = 0$$

On comparing with

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$2g = -8 \ ; \quad 2f = -8 \ ; \quad c = 22$$

$$g = -4 \ ; \quad f = -4 \ ; \quad c = 22$$

$$C \equiv (-g , -f) \qquad R = \sqrt{g^2 + f^2 - c}$$

$$\equiv (4,4) \qquad = \sqrt{16 + 16 - 22}$$

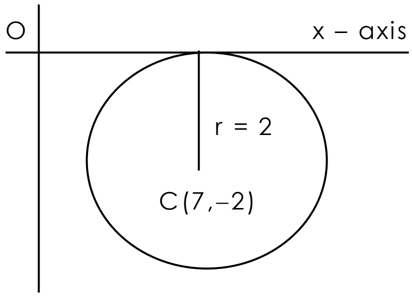
$$= \sqrt{10}$$

Q3.

01. find equation of the circle having centre (7,–2) and touching the x – axis

SOLUTION

$r = 2$ …… (REFER DIAGRAM)



$C(7,-2)$, $r = 2$

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 7)^2 + (y + 2)^2 = (2)^2$$

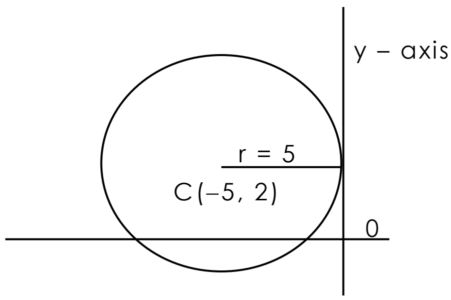
$$x^2 - 14x + 49 + y^2 + 4y + 4 = 4$$

$x^2 + y^2 - 14x + 4y + 49 = 0$ …….. equation of the circle

02. find equation of the circle having centre (–5,2) and touching the y – axis

SOLUTION

$r = 5$ …… (REFER DIAGRAM)



$C(-5, 2)$, $r = 5$

$$(x - h)^2 + (y - k)^2 = r^2$$

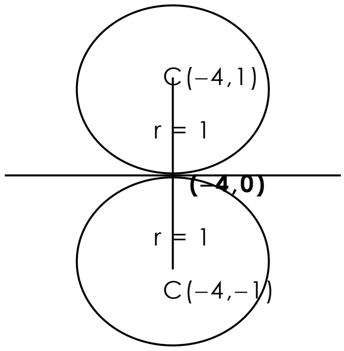
$$(x + 5)^2 + (y - 2)^2 = (5)^2$$

$$x^2 + 10x + 25 + y^2 - 4y + 4 = 25$$

$x^2 + y^2 + 10x - 4y + 4 = 0$ …….. equation of the circle

03. find equation of the circle having radius = 1 and touching the x – axis at (–4,0)

SOLUTION



CIRCLE 1 $C(-4,1)$, $r = 1$

$(x - h)^2 + (y - k)^2 = r^2$

$(x + 4)^2 + (y - 1)^2 = 1$

$x^2 + 8x + 16 + y^2 - 2y + 1 = 1$

$x^2 + y^2 + 8x - 2y + 16 = 0$

CIRCLE 2 $C(-4,-1)$, $r = 1$

$(x - h)^2 + (y - k)^2 = r^2$

$(x + 4)^2 + (y + 1)^2 = 1$

$x^2 + 8x + 16 + y^2 + 2y + 1 = 1$

$x^2 + y^2 + 8x + 2y + 16 = 0$

ans : $x^2 + y^2 + 8x \pm 2y + 16 = 0$

Q4.

01. Find circle touching both the axes and having radius 7

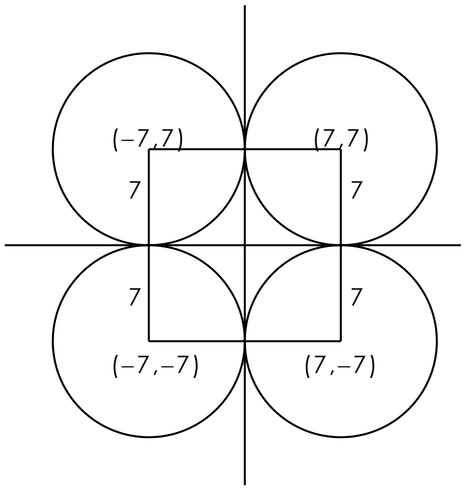
SOLUTION

Using : $(x - h)^2 + (y - k)^2 = r^2$

CIRCLE 1 $C(7,7)$, $r = 7$

$(x - 7)^2 + (y - 7)^2 = 7^2$

$x^2 + y^2 - 14x - 14y + 49 = 0$



CIRCLE 2 $C(-7,7)$, $r = 7$

$(x + 7)^2 + (y - 7)^2 = 7^2$

$x^2 + y^2 + 14x - 14y + 49 = 0$

CIRCLE 3 $C(-7,-7)$, $r = 7$

$(x + 7)^2 + (y + 7)^2 = 7^2$

$x^2 + y^2 + 14x + 14y + 49 = 0$

CIRCLE 4 $C(7,-7)$, $r = 7$

$(x - 7)^2 + (y + 7)^2 = 7^2$

$x^2 + y^2 - 14x + 14y + 49 = 0$

ans : $x^2 + y^2 \pm 14x \pm 14y + 49 = 0$

02. find equation of the circle touching both axes and passing through (1,2)

SOLUTION

STEP 1 : $CP = r$

$CP^2 = r^2$

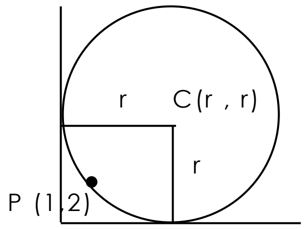
$(r-1)^2 + (r-2)^2 = r^2$

$r^2 - 2r + 1 + r^2 - 4r + 4 = r^2$

$r^2 - 6r + 5 = 0$

$(r-1)(r-5) = 0$

$r = 1 \;;\; r = 5$



STEP 2 : $r = 1 \;;\; C(1,1)$

$(x-h)^2 + (y-k)^2 = r^2$

$(x-1)^2 + (y-1)^2 = 1$

$x^2 - 2x + 1 + y^2 - 2y + 1 = 1$

$x^2 + y^2 - 2x - 2y + 1 = 0$

$r = 5 \;;\; C(5,5)$

$(x-h)^2 + (y-k)^2 = r^2$

$(x-5)^2 + (y-5)^2 = 25$

$x^2 - 10x + 25 + y^2 - 10y + 25 = 25$

$x^2 + y^2 - 10x - 10y + 25 = 0$

03. find equation of the circle touching both axes and passing through (−9,8)

SOLUTION

STEP 1 : $CP = r$

$CP^2 = r^2$

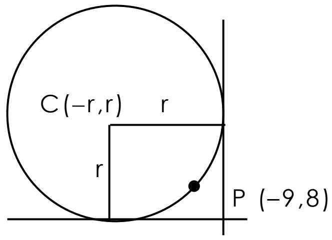
$(-r+9)^2 + (r-8)^2 = r^2$

$r^2 - 18r + 81 + r^2 - 16r + 64 = r^2$

$r^2 - 34r + 145 = 0$

$(r-5)(r-29) = 0$

$r = 5 \;;\; r = 29$



STEP 2 : $r = 5 \;;\; C(-5,5)$

$(x-h)^2 + (y-k)^2 = r^2$

$(x+5)^2 + (y-5)^2 = 25$

$x^2 + 10x + 25 + y^2 - 10y + 25 = 25$

$x^2 + y^2 + 10x - 10y + 25 = 0$

$r = 29 \;;\; C(-29,29)$

$(x-h)^2 + (y-k)^2 = r^2$

$(x+29)^2 + (y-29)^2 = 841$

$x^2 + 58x + 841 + y^2 - 58y + 841 = 841$

$x^2 + y^2 + 58x - 58y + 841 = 0$

Q5.

01. Find equation of circle with center (4,3) & touching $5x - 12y - 10 = 0$

SOLUTION

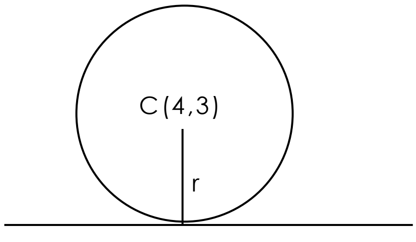
STEP 1 :

$$r = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

$$= \left|\frac{5(4) - 12(3) - 10}{\sqrt{5^2 + 12^2}}\right|$$

$$= \left|\frac{20 - 36 - 10}{\sqrt{169}}\right|$$

$$= \left|\frac{-26}{13}\right| = 2$$



STEP 2 :

$C(4,3) \quad , \quad r = 2$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 4)^2 + (y - 3)^2 = (2)^2$

$x^2 - 8x + 16 + y^2 - 6y + 9 = 4$

$x^2 + y^2 - 8x - 6y + 25 - 4 = 0$

$x^2 + y^2 - 8x - 6y + 21 = 0$ …….. equation of the circle

02. Find equation of circle with center (3,1) & touching $8x - 15y + 25 = 0$

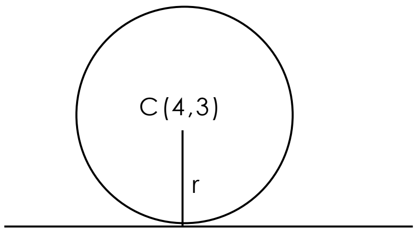
STEP 1 :

$$r = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

$$= \left|\frac{8(3) - 15(1) + 25}{\sqrt{8^2 + 15^2}}\right|$$

$$= \left|\frac{24 - 15 + 25}{\sqrt{289}}\right|$$

$$= \left|\frac{34}{17}\right| = 2$$



STEP 2 :

$C(3,1) \quad , \quad r = 2$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 3)^2 + (y - 1)^2 = (2)^2$

$x^2 - 6x + 9 + y^2 - 2y + 1 = 4$

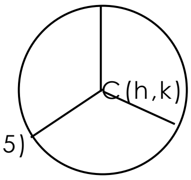
$x^2 + y^2 - 6x - 2y + 10 - 4 = 0$

$x^2 + y^2 - 6x - 2y + 6 = 0$ …….. equation of the circle

Q6.

01. Find equation of circle passing through (4 , 6) ; (–3 , 5) & (5 , –1)

SOLUTION



STEP 1 :

$CP = CQ$

$CP^2 = CQ^2$

$(h-4)^2 + (k-6)^2 = (h+3)^2 + (k-5)^2$

$h^2 - 8h + 16 + k^2 - 12k + 36 = h^2 + 6h + 9 + k^2 - 10k + 25$

$-8h - 12k + 52 = 6h - 10k + 34$

$18 = 14h + 2k$

$9 = 7h + k$ ……………… (1)

STEP 2 :

$CP = CR$

$CP^2 = CR^2$

$(h-4)^2 + (k-6)^2 = (h-5)^2 + (k+1)^2$

$h^2 - 8h + 16 + k^2 - 12k + 36 = h^2 - 10h + 25 + k^2 + 2k + 1$

$-8h - 12k + 52 = -10h + 2k + 26$

$2h - 14k = -26$

$h - 7k = -13$ ………………… (2)

STEP 3 : SOLVING (1) & (2)

$7 \times \quad 7h + k = 9$

$h - 7k = -13$

$49h + 7k = 63$

$h - 7k = -13$

$50h = 50$

$h = 1$

$k = 2$ $\quad C \equiv (1, 2)$

STEP 4 : C (1 , 2) , P(4,6)

$r = CP$

$= \sqrt{(1-4)^2 + (2-6)^2}$

$= \sqrt{25}$

$= 5$

STEP 5 : C(1 , 2) , r = 5

$(x-h)^2 + (y-k)^2 = r^2$

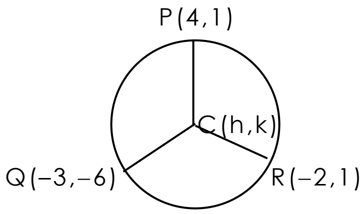
$(x-1)^2 + (y-2)^2 = 5^2$

$x^2 - 2x + 1 + y^2 - 4y + 4 = 25$

$x^2 + y^2 - 2x - 4y - 20 = 0$ ………. Equation of circle

02. Find equation of circle passing through (4 , 1) ; (−3 , −6) & (−2 , 1)

SOLUTION



STEP 1 :

$CP = CQ$

$CP^2 = CQ^2$

$(h-4)^2 + (k-1)^2 = (h+3)^2 + (k+6)^2$

$h^2 - 8h + 16 + k^2 - 2k + 1 = h^2 + 6h + 9 + k^2 + 12k + 36$

$-8h - 2k + 17 = 6h + 12k + 45$

$-28 = 14h + 14k$

$-2 = h + k$ ……………… (1)

STEP 2 :

$CP = CR$

$CP^2 = CR^2$

$(h-4)^2 + (k-1)^2 = (h+2)^2 + (k-1)^2$

$h^2 - 8h + 16 + k^2 - 2k + 1 = h^2 + 4h + 4 + k^2 - 2k + 1$

$-8h - 2k + 17 = 4h - 2k + 5$

$12 = 12h$

$h = 1$ ………………… (2)

STEP 3 : Solving (1) & (2)

sub $h = 1$ in (1)

$k = -3$ $C \equiv (1, -3)$

STEP 4 : C (1 , −3) , P(4,1)

$r = CP$

$= \sqrt{(1-4)^2 + (-3-1)^2}$

$= \sqrt{9 + 16}$

$= 5$

STEP 5 : C(1 , −3) , r = 5

$(x-h)^2 + (y-k)^2 = r^2$

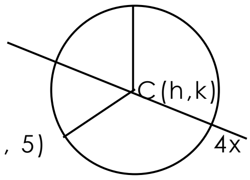
$(x-1)^2 + (y+3)^2 = 25$

$x^2 - 2x + 1 + y^2 + 6y + 9 = 25$

$x^2 + y^2 - 2x + 6y - 15 = 0$ ………. Equation of circle

05. Find equation of circle passing through (4 , 1) ; (6 , 5) & whose center lies on $4x + y = 16$

SOLUTION



STEP 1 :

Since C(h , k) lies on $4x + y = 16$

$4h + k = 16$ ……………… (1)

STEP 2 :

$CP = CQ$

$CP^2 = CQ^2$

$(h - 4)^2 + (k - 1)^2 = (h - 6)^2 + (k - 5)^2$

$h^2 - 8h + 16 + k^2 - 2k + 1 = h^2 - 12h + 36 + k^2 - 10k + 25$

$-8h - 2k + 17 = -12h - 10k + 61$

$4h + 8k = 44$

$h + 2k = 11$ ………………… (2)

STEP 3 : $2 \times \quad 4h + k = 16$

$h + 2k = 11$

$8h + 2k = 32$

$h + 2k = 11$

$7h = 21$

$h = 3$

$k = 4$ $\quad C \equiv (3 , 4)$

STEP 4 : C (3 , 4) , P(4,1)

$r = CP$

$= \sqrt{(3 - 4)^2 + (4 - 1)^2}$

$= \sqrt{1 + 9}$

$= \sqrt{10}$

STEP 5 : C(3 , 4) , $r = \sqrt{10}$

$(x - h)^2 + (y - k)^2 = r^2$

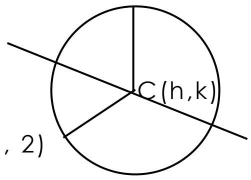
$(x - 3)^2 + (y - 4)^2 = 10$

$x^2 - 6x + 9 + y^2 - 8y + 16 = 10$

$x^2 + y^2 - 6x - 8y + 15 = 0$ ………. Equation of circle

06. Find equation of circle passing through (1 , –4) ; (5 , 2) & whose center lies on $x - 2y + 9 = 0$

SOLUTION



STEP 1 :

Since C(h , k) lies on $x - 2y + 9 = 0$

$h - 2k = -9$ ……………… (1)

STEP 2 :

$CP = CQ$

$CP^2 = CQ^2$

$(h-1)^2 + (k+4)^2 = (h-5)^2 + (k-2)^2$

$h^2 - 2h + 1 + k^2 + 8k + 16 = h^2 - 10h + 25 + k^2 - 4k + 4$

$-2h + 8k + 17 = -10h - 4k + 29$

$8h + 12k = 12$

$2h + 3k = 3$ ………………… (2)

STEP 3 :

$2 \times \quad h - 2k = -9 \qquad\qquad 2h - 4k = -18$

$2h + 3k = 3 \qquad\qquad 2h + 3k = 3$

$-7k = -21$

$k = 3$

subs in (1) $h = -3$ $\qquad C \equiv (-3 , 3)$

STEP 4 : C (–3 , 3) , P(1,–4)

$r = CP$

$= \sqrt{(-3-1)^2 + (3+4)^2}$

$= \sqrt{16 + 49}$

$= \sqrt{65}$

STEP 5 : C(–3 , 3), $r = \sqrt{65}$

$(x-h)^2 + (y-k)^2 = r^2$

$(x+3)^2 + (y-3)^2 = 65$

$x^2 + 6x + 9 + y^2 - 6y + 9 = 65$

$x^2 + y^2 + 6x - 6y + 18 - 65 = 0$

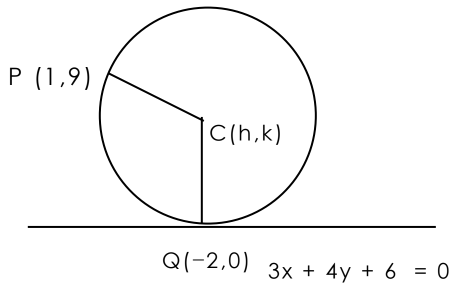
$x^2 + y^2 + 6x - 6y - 47 = 0$ ………. Equation of circle

Q7.

01. find equation of circle passing through (1 , 9) & touching $3x + 4y + 6 = 0$ at $(-2 , 0)$

SOLUTION

STEP 1 :



$CP = CQ$

$CP^2 = CQ^2$

$(h - 1)^2 + (k - 9)^2 = (h + 2)^2 + (k - 0)^2$

$h^2 - 2h + 1 + k^2 - 18k + 81 = h^2 + 4h + 4 + k^2$

$-2h - 18k + 82 = 4h + 4$

$78 = 6h + 18k$

$h + 3k = 13$ ………………… (1)

STEP 2 :

Slope of line

$3x + 4y + 6 = 0$: $m = -\frac{a}{b} = -\frac{3}{4}$

$\therefore\ m_{CQ} = \frac{4}{3}$ ……. (Tangent – Radius)

Now ; $m_{CQ} = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{4}{3} = \frac{k - 0}{h + 2}$

$4h + 8 = 3k$

$4h - 3k = -8$ ………. (2)

STEP 3 :

Solving (1) & (2)

$h + 3k = 13$

$4h - 3k = -8$

$5h = 5$

$h = 1$

sub in (1) , $k = 4$

C(1,4)

STEP 4 : $r = CQ$, C(1,4) , Q(−2,0)

$= \sqrt{(1 + 2)^2 + (4 - 0)^2}$

$= 5$

STEP 5 : C(1 , 4) , r = 5

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 1)^2 + (y - 4)^2 = 25$

$x^2 - 2x + 1 + y^2 - 8y + 16 = 25$

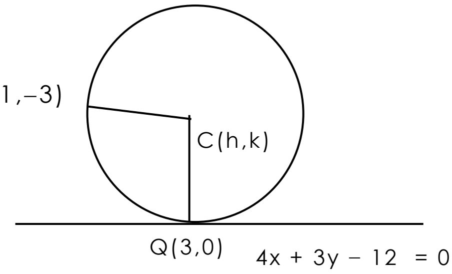
$x^2 + y^2 - 2x - 8y + 17 - 25 = 0$

$x^2 + y^2 - 2x - 8y - 8 = 0$ ………. Equation of circle

02. find equation of circle passing through $(-1, -3)$ & touching $4x + 3y - 12 = 0$ at $(3, 0)$

SOLUTION

STEP 1 :



$CP = CQ$

$CP^2 = CQ^2$

$(h + 1)^2 + (k + 3)^2 = (h - 3)^2 + (k - 0)^2$

$h^2 + 2h + 1 + k^2 + 6k + 9 = h^2 - 6h + 9 + k^2$

$2h + 6k + 10 = -6h + 9$

$8h + 6k = -1$ .... (1)

STEP 2 :

Slope of line

$4x + 3y - 12 = 0 \;:\; m = -\frac{a}{b} = -\frac{4}{3}$

$\therefore\ m_{CQ} = \frac{3}{4}$ .... (Tangent – Radius)

Now ; $m_{CQ} = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{3}{4} = \frac{k - 0}{h - 3}$

$3h - 9 = 4k$

$3h - 4k = 9$ .... (2)

STEP 3 :

Solving (1) & (2)

(1) x 2 $\quad 16h + 12k = -2$

(2) x 3 $\quad 9h - 12k = 27$

$25h = 25$

$h = 1$

sub in (1) , $k = \frac{-3}{2}$

C $(1, -3/2)$

STEP 4 : $r = CQ$, $C(1,-3/2)$, $Q(3,0)$

$= \sqrt{(1 - 3)^2 + (-3/2 - 0)^2}$

$= \sqrt{4 + \frac{9}{4}} = \sqrt{\frac{25}{4}} = \frac{5}{2}$

STEP 5 : $C(1, -3/2)$, $r = 5/2$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 1)^2 + (y + 3/2)^2 = 25/4$

$x^2 - 2x + 1 + y^2 + 3y + \frac{9}{4} = \frac{25}{4}$

$\frac{4x^2 - 8x + 4 + 4y^2 + 12y + 9}{4} = \frac{25}{4}$

$4x^2 + 4y^2 - 8x + 12y + 13 - 25 = 0$

$4x^2 + 4y^2 - 8x + 12y - 12 = 0$

$x^2 + y^2 - 2x + 3y - 3 = 0$ .... Equation of circle

Q8.

01. Find equation of circle with center (3 , –1) and which cuts off a chord of length 6 on line $2x - 5y + 18 = 0$

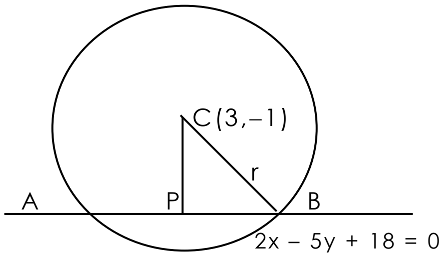
SOLUTION

STEP 1 : AP = PB = 3 (⊥ from the centre bisects the chord)

STEP 2 :

$$CP = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

$$= \left| \frac{2(3) - 5(-1) + 18}{\sqrt{2^2 + 5^2}} \right|$$



$$= \left| \frac{6 + 5 + 18}{\sqrt{29}} \right|$$

$$= \left| \frac{29}{\sqrt{29}} \right|$$

$$CP = \sqrt{29}$$

STEP 3 : In Δ CPB ; $CP^2 + PB^2 = r^2$

$$29 + 9 = r^2$$

$$r^2 = 38$$

$$r = \sqrt{38}$$

STEP 4 : $C(3 , -1), r = \sqrt{38}$

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 3)^2 + (y + 1)^2 = 38$$

$$x^2 - 6x + 9 + y^2 + 2y + 1 = 38$$

$$x^2 + y^2 - 6x + 2y + 10 - 38 = 0$$

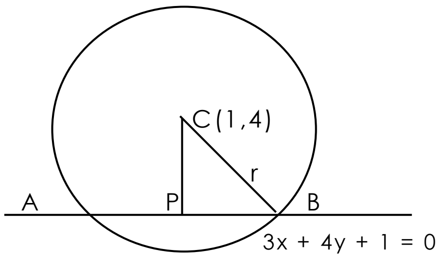
$x^2 + y^2 - 6x + 2y - 28 = 0$ ………. Equation of circle

02. Find equation of circle with center (1 , 4) and which cuts off a chord of length 6 on line $3x + 4y + 1 = 0$

SOLUTION

STEP 1 : $AP = PB = 3$ ($\perp$ from the centre bisects the chord)

STEP 2 : $CP = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$



$= \left| \frac{3(1) + 4(4) + 1}{\sqrt{3^2 + 4^2}} \right|$

$= \left| \frac{3 + 16 + 1}{\sqrt{25}} \right|$

$= \left| \frac{20}{5} \right|$

$CP = 4$

STEP 3 : In Δ CPB ; $CP^2 + PB^2 = r^2$

$16 + 9 = r^2$

$r^2 = 25$

$r = 5$

STEP 4 : C(1 , 4), r = 5

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 1)^2 + (y - 4)^2 = 25$

$x^2 - 2x + 1 + y^2 - 8y + 16 = 25$

$x^2 + y^2 - 2x - 8y + 17 - 25 = 0$

$x^2 + y^2 - 2x - 8y - 8 = 0$ ……… Equation of circle

03. Find the length of intercept made by circle $x^2 + y^2 - 2x - 8y - 8 = 0$ on the line $3x + 4y + 1 = 0$

SOLUTION

STEP 1 : $x^2 + y^2 - 2x - 8y - 8 = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -2$; $2f = -8$

$g = -1$; $f = -4$; $c = -8$

$C \equiv (-g, -f)$ $\qquad$ $r = \sqrt{g^2 + f^2 - c}$

$\equiv (1, 4)$ $\qquad$ $= \sqrt{1 + 16 + 8}$

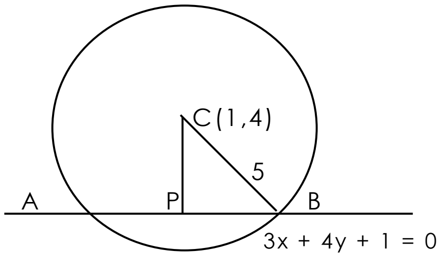
$= 5$

STEP 2 : $CP = \left|\dfrac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$

$= \left|\dfrac{3(1) + 4(4) + 1}{\sqrt{3^2 + 4^2}}\right|$

$= \left|\dfrac{3 + 16 + 1}{\sqrt{25}}\right|$

$= \left|\dfrac{20}{5}\right|$



$CP = 4$

STEP 3 : In Δ CPB ; $CP^2 + PB^2 = r^2$

$16 + PB^2 = 25$

$PB^2 = 9$

$PB = 3$

STEP 4 : $AB = 2(PB) = 6$ .... ($\perp$ from the centre bisects the chord)

04. Find the length of intercept made by circle $x^2 + y^2 - 6x + 4y - 12 = 0$ on the line $4x - 3y + 2 = 0$

SOLUTION

STEP 1 :

$x^2 + y^2 - 6x + 4y - 12 = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -6 ; \quad 2f = 4$

$g = -3 ; \quad f = 2 \quad ; \quad c = -12$

$C \equiv (-g, -f) \qquad r = \sqrt{g^2 + f^2 - c}$

$\equiv (3, -2) \qquad = \sqrt{9 + 4 + 12}$

$= 5$

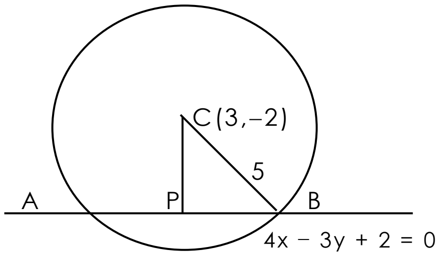
STEP 2 :

$CP = \left| \dfrac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$

$= \left| \dfrac{4(3) - 3(-2) + 2}{\sqrt{3^2 + 4^2}} \right|$

$= \left| \dfrac{12 + 6 + 2}{\sqrt{25}} \right|$

$= \left| \dfrac{20}{5} \right|$



$CP = 4$

STEP 3 :

In Δ CPB ; $CP^2 + PB^2 = r^2$

$16 + PB^2 = 25$

$PB^2 = 9$

$PB = 3$

STEP 4 :

$AB = 2(PB) = 6$ .... ($\perp$ from the centre bisects the chord)

MARCH – 2017

Find equation of circle passing through point of intersection of the lines $x + 3y = 0$ and $2x - 7y = 0$ and whose centre is the point of intersection of the lines $x + y + 1 = 0$ and $x - 2y + 4 = 0$

SOLUTION

STEP 1

point of intersection of the lines $x + 3y = 0$ and $2x - 7y = 0 \equiv (0,0)$

STEP 2

point of intersection of the lines $x + y + 1 = 0$ and $x - 2y + 4 = 0 \equiv (-2,1)$

STEP 3

Centre of the circle is $(-2,1)$ and circle passes through $(0,0)$

$$\text{Radius } r = \sqrt{(-2-0)^2 + (1-0)^2}$$

$$= \sqrt{4+1}$$

$$= \sqrt{5}$$

STEP 4 Equation of the circle

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x+2)^2 + (y-1)^2 = 5$$

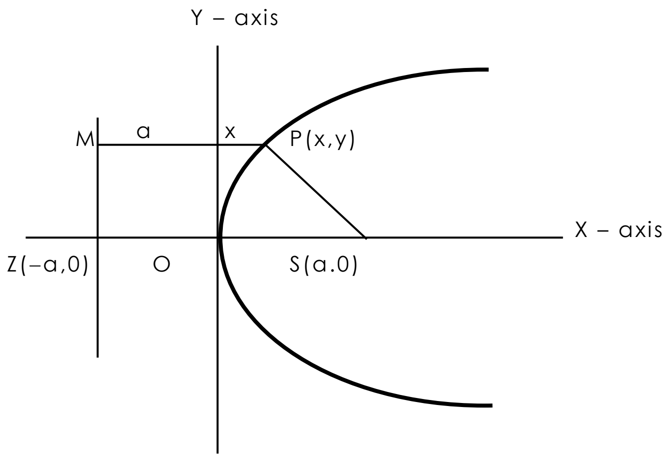
$$x^2 + y^2 + 4x - 2y = 0$$

PARABOLA

STANDARD **PARABOLA :** $\mathbf{y^2 = 4ax}$

VERTEX – O(0,0)

AXIS OF SYMMETRY – POSITIVE X – AXIS



Using Focus Directrix property

$$PS = PM$$

$$PS^2 = PM^2$$

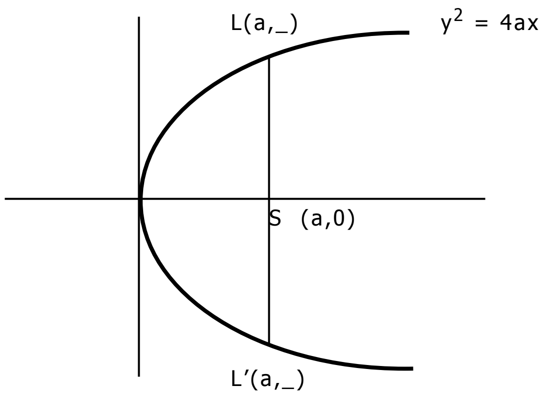
$$(x - a)^2 + y^2 = (x + a)^2$$

$$x^2 - 2ax + a^2 + y^2 = x^2 + 2ax + a^2$$

$$y^2 = 4ax$$

.... equation of the parabola

END POINTS AND LENGTH OF THE LATUS RECTUM



Since L and L' lie on the parabola $y^2 = 4ax$ it must satisfy the equation of the parabola

$\therefore$ sub $x = a$ in $y^2 = 4ax$

$y^2 = 4a^2$

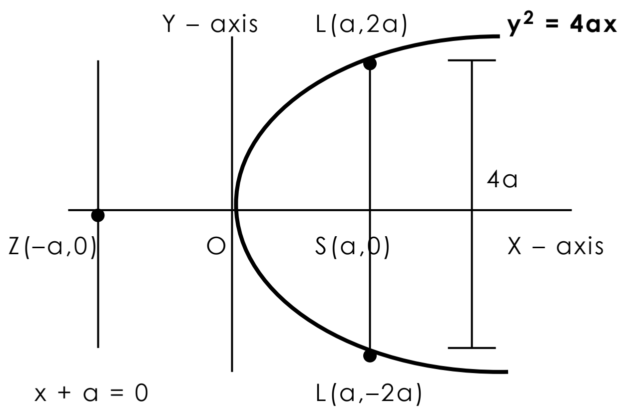
$y = \pm 2a$

Hence $L \equiv (a , 2a)$ and $L' \equiv (a , -2a)$;

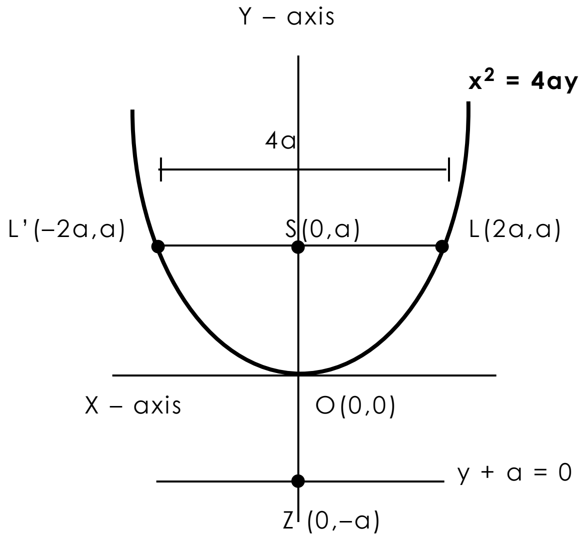
Length LL' = $2a + 2a = 4a$

STANDARD FORMS OF PARABOLA

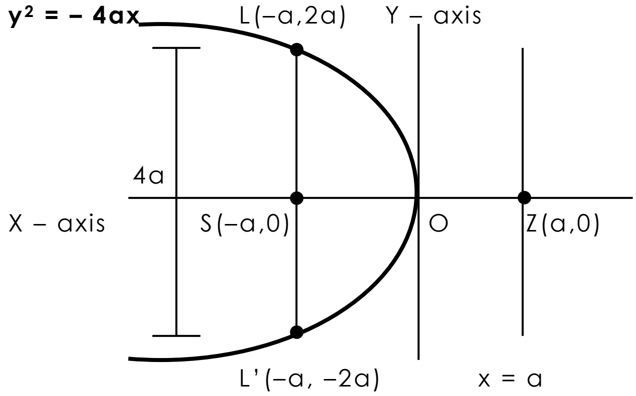
PARABOLA : $y^2 = 4ax$



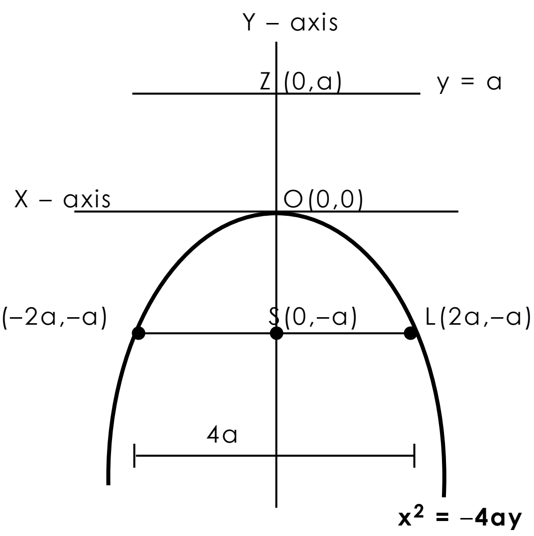
PARABOLA : $x^2 = 4ay$



PARABOLA : $y^2 = -4ax$



PARABOLA : $x^2 = -4ay$



Q. SET

01. find equation of parabola having focus at (3,0) and directrix $x + 3 = 0$

ans : $y^2 = 12x$

02. find equation of parabola having focus at (5,0) and directrix $x + 5 = 0$

ans : $y^2 = 20x$

03. find equation of parabola having(3 , 6) and (3 , –6) as extremities of latus rectum .

ans : $y^2 = 12x$

04. find equation of parabola having(4 , 8) and (4 , –8) as extremities of latus rectum .

ans : $y^2 = 16x$

05. the ordinate of point P on the parabola $y^2 = 8x$ is 4 . Find focal distance of point P

ans : 4 units

06. the ordinate of point P on the parabola $y^2 = 16x$ is 8 . Find focal distance of point P

ans : 8 units

07. find focal distance of point P on the parabola $5y^2 = 12x$ if the abscissa of P is equal to 7

ans : $^{38}/_{5}$ units

08. find coordinates of point on parabola $y^2 = 32x$ having focal distance 10

ans : (2,8) & (2,–8)

09. find coordinates of point on parabola $y^2 = 4x$ having focal distance 5

ans : (4,4) & (4,–4)

10. line $y = x - 8$ intersects parabola $y^2 = 4x$ in A and B . Find the length of the chord AB

ans : $12\sqrt{2}$ units

11. find length of intercept made by the parabola $y^2 = 4x$ on $x + y = 3$

ans : $8\sqrt{2}$ units

12. Find the equation of the parabola **(Using Definition)** whose focus is (5,0) and equation of the directrix is $x = -5$

ans : $y^2 = 20x$

13. Find the equation of the parabola **(Using Definition)** whose focus is (0,5) and equation of the directrix is $y = -5$

ans : $x^2 = 20y$

14. Find coordinates of focus ; equation of directrix ; length of latus rectum and ends of latus rectum

(1) $y^2 = 12x$ (2) $y^2 + 16x = 0$ (3) $5y^2 + 16x = 0$ (4) $x^2 = 8y$

(5) $x^2 + 12y = 0$

SOLUTION SET

01. find equation of parabola having focus at (3,0) and directrix $x + 3 = 0$

SOLUTION :

Let the equation of the parabola : $y^2 = 4ax$

Focus : $S \equiv (a, 0) \equiv (3,0)$ .... given $\therefore a = 3$

Directrix : $x + a = 0$

$x + 3 = 0$ .... Given $\therefore a = 3$

Hence equation of the parabola : $y^2 = 4ax$

$y^2 = 4(3)x$

$y^2 = 12x$

02. find equation of parabola having focus at (5,0) and directrix $x + 5 = 0$

SOLUTION :

Let the equation of the parabola : $y^2 = 4ax$

Focus : $S \equiv (a, 0) \equiv (5,0)$ .... given $\therefore a = 5$

Directrix : $x + a = 0$

$x + 5 = 0$ .... Given $\therefore a = 5$

Hence equation of the parabola : $y^2 = 4ax$

$y^2 = 4(5)x$

$y^2 = 20x$

03. find equation of parabola having(3 , 6) and (3 , –6) as extremities of latus rectum .

SOLUTION :

Let the equation of the parabola : $y^2 = 4ax$

Extremities of Latus – Rectum : $L \equiv (a,2a) \equiv (3,6)$

$L' \equiv (a,-2a) \equiv (3,-6)$ .... Given

On comparing : $a = 3$

Hence equation of the parabola : $y^2 = 4ax$

$y^2 = 4(3)x$

$y^2 = 12x$

04. find equation of parabola having(4 , 8) and (4 , –8) as extremities of latus rectum .

SOLUTION :

Let the equation of the parabola : $y^2 = 4ax$

Extremities of Latus – Rectum : $L \equiv (a,2a) \equiv (4,8)$

$L' \equiv (a,-2a) \equiv (4,-8)$ .... Given

On comparing : $a = 4$

Hence equation of the parabola : $y^2 = 4ax$

$y^2 = 4(4)x$

$y^2 = 16x$

05. the ordinate of point P on the parabola $y^2 = 8x$ is 4 . Find focal distance of point P

SOLUTION :

STEP 1 : Equation of parabola : $y^2 = 8x$

On comparing with : $y^2 = 4ax$, $4a = 8$, $a = 2$

STEP 2 : P(x , 4) lies on parabola $y^2 = 8x$ & hence it must satisfy the equation of the parabola

$\therefore 4^2 = 8x$

$x = 2$, P(2,4)

STEP 3 : Focal distance of point P(2,4) = PS

= PM ………. FOCUS DIRECTRIX PROPERTY

= x + a

= 2 + 2 = 4 units

06. the ordinate of point P on the parabola $y^2 = 16x$ is 8 . Find focal distance of point P

SOLUTION :

STEP 1 : Equation of parabola : $y^2 = 16x$

On comparing with : $y^2 = 4ax$, $4a = 16$, $a = 4$

STEP 2 : P(x , 8) lies on parabola $y^2 = 16x$ & hence it must satisfy the equation of parabola

$\therefore 8^2 = 16x$

$x = 4$, P(4,8)

STEP 3 : Focal distance of point P(4,8) = PS

= PM ………. FOCUS DIRECTRIX PROPERTY

= x + a

= 4 + 4 = 8 units

07. find focal distance of point P on the parabola $5y^2 = 12x$ if the abscissa of P is equal to 7

SOLUTION :

STEP 1 : Equation of parabola : $5y^2 = 12x$

$$y^2 = \frac{12}{5}x$$

On comparing with : $y^2 = 4ax$, $4a = \frac{12}{5}$, $a = \frac{3}{5}$

STEP 2 : Focal distance of point P(7,y) = PS

= PM ………. FOCUS DIRECTRIX PROPERTY

= x + a

$= 7 + \frac{3}{5} = \frac{38}{5}$ units

08. find coordinates of point on parabola $y^2 = 32x$ having focal distance 10

SOLUTION :

STEP 1 : Equation of parabola : $y^2 = 32x$

On comparing with : $y^2 = 4ax$, $4a = 32$, $a = 8$

STEP 2 : Focal distance of point P(x,y) = 10

PS = 10

PM = 10 ………. FOCUS DIRECTRIX PROPERTY

x + a = 10

x + 8 = 10   x = 2

STEP 3 : P(2 , y) lies on parabola $y^2 = 32x$ & hence it must satisfy the equation of parabola

$\therefore y^2 = 16(2)$

$y^2 = 64$,

$y = \pm 8$ $\therefore$ P(2,8) & P'(2,–8)

(NOTE : THERE ARE ALWAYS 2 POINTS ON THE PARABOLA HAVING SAME FOCAL DISTANCE FROM THE FOCUS)

09. find coordinates of point on parabola $y^2 = 4x$ having focal distance 5

SOLUTION :

STEP 1 : Equation of parabola : $y^2 = 4x$

On comparing with : $y^2 = 4ax$, $4a = 4$, $a = 1$

STEP 2 : Focal distance of point P(x,y) = 5

$$PS = 5$$

$$PM = 5 \quad \text{.......... FOCUS DIRECTRIX PROPERTY}$$

$$x + a = 5$$

$$x + 1 = 5 \qquad x = 4$$

STEP 3 : P(4 , y) lies on parabola $y^2 = 4x$ & hence it must satisfy the equation of parabola

$$\therefore\ y^2 = 4(4)$$

$$y^2 = 16,$$

$$y = \pm 4 \qquad \therefore\ P(4,4)\ \&\ P'(4,-4)$$

10. line $y = x - 8$ intersects parabola $y^2 = 4x$ in A and B . Find the length of the chord AB

SOLUTION :

STEP 1 : equation the line : $y = x - 8$ .... (1)

Equation of parabola : $y^2 = 4x$ .... (2)

Line intersects parabola in A and B .

Solving (1) & (2) : $(x - 8)^2 = 4x$

$$x^2 - 16x + 64 = 4x$$

$$x^2 - 20x + 64 = 0$$

$$(x - 4)(x - 16) = 0$$

$x = 4$ | $x = 16$

subs in (1)

$y = 4 - 8$ | $y = 16 - 8$

$y = -4$ | $y = 8$

A(4,−4) | B(16,8)

STEP 2 : length of chord AB $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$$= \sqrt{(4 - 16)^2 + (-4 - 8)^2}$$

$$= \sqrt{(-12)^2 + (-12)^2}$$

$$= \sqrt{144 + 144}$$

$$= \sqrt{288} = 12\sqrt{2} \text{ units}$$

11. find length of intercept made by the parabola $y^2 = 4x$ on $x + y = 3$

SOLUTION :

STEP 1 : equation the line : $y = 3 - x$ .... (1)

Equation of parabola : $y^2 = 4x$ .... (2)

Line intersects parabola in A and B .

Solving (1) & (2) : $(3 - x)^2 = 4x$

$9 - 6x + x^2 = 4x$

$x^2 - 10x + 9 = 0$

$(x - 1)(x - 9) = 0$

$x = 1$ | $x = 9$

subs in (1)

$y = 3 - 1$ | $y = 3 - 9$

$y = 2$ | $y = -6$

A(1,2) | B(9,–6)

STEP 2 : length of chord AB $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(1 - 9)^2 + (2 + 6)^2}$

$= \sqrt{(-8)^2 + (8)^2}$

$= \sqrt{64 + 64}$

$= \sqrt{128} = 8\sqrt{2}$ units

12. Find the equation of the parabola **(Using Definition)** whose focus is (5,0) and equation of the directrix is $x = -5$

Let P(x,y) be any point on the parabola

By focus directrix property

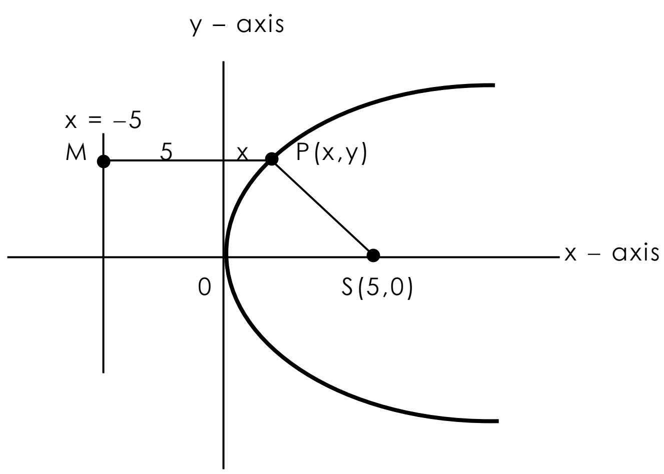
$PS = PM$

$PS^2 = PM^2$

$(x - 5)^2 + (y - 0)^2 = (x + 5)^2$

$x^2 - 10x + 25 + y^2 = x^2 + 10x + 25$

$y^2 = 20x$



13. Find the equation of the parabola **(Using Definition)** whose focus is (0,5) and equation of the directrix is $y = -5$

Let P(x,y) be any point on the parabola

By focus directrix property

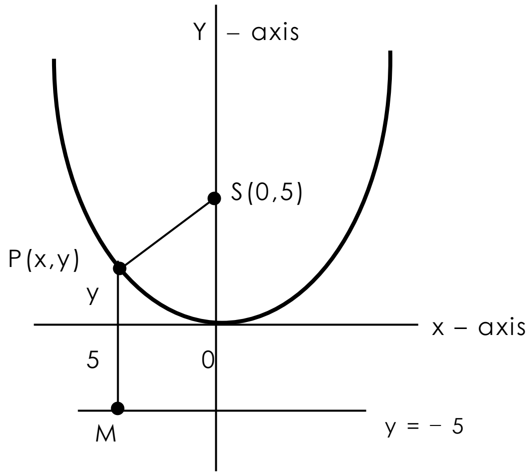
$PS = PM$

$PS^2 = PM^2$

$(x-0)^2 + (y-5)^2 = (y+5)^2$

$x^2 + y^2 - 10y + 25 = y^2 + 10y + 25$

$x^2 = 20y$



14. Find coordinates of focus ; equation of directrix ; length of latus rectum and ends of latus rectum

(1) $y^2 = 12x$

SOLUTION :

Equation of parabola	:	$y^2 = 12x$
On comparing with	:	$y^2 = 4ax$, $4a = 12$, $a = 3$
Focus	:	$S \equiv (a,0) \equiv (3,0)$
Equation of directrix	:	$x + a = 0 \therefore x + 3 = 0$
Length of latus recturm	=	$4a = 12$ units
Ends of latus rectum	:	$L \equiv (a, 2a) \equiv (3,6)$
		$L' \equiv (a,-2a) \equiv (3,-6)$

(2) $y^2 + 16x = 0$

$y^2 = -16x$

SOLUTION :

Equation of parabola	:	$y^2 = -16x$
On comparing with	:	$y^2 = -4ax$, $4a = 16$, $a = 4$
Focus	:	$S \equiv (-a,0) \equiv (-4,0)$
Equation of directrix	:	$x - a = 0 \therefore x - 4 = 0$
Length of latus recturm	=	$4a = 16$ units
Ends of latus rectum	:	$L \equiv (-a, 2a) \equiv (-4,8)$
		$L' \equiv (-a,-2a) \equiv (-4,-8)$

(3) $5y^2 + 16x = 0$

$5y^2 = -16x$

SOLUTION :

Equation of parabola : $y^2 = \frac{-16}{5}x$

On comparing with : $y^2 = -4ax$, $4a = \frac{16}{5}$, $a = \frac{4}{5}$

Focus : $S \equiv (-a,0) \equiv (-4/5,0)$

Equation of directrix : $x - a = 0 \therefore x - \frac{4}{5} = 0$

Length of latus recturm $= 4a = \frac{16}{5}$ units

Ends of latus rectum : $L \equiv (-a , 2a) \equiv (-4/5, 8/5)$

$L' \equiv (-a,-2a) \equiv (-4/5,-8/5)$

(4) $x^2 = 8y$

SOLUTION :

Equation of parabola : $x^2 = 8y$

On comparing with : $x^2 = 4ay$, $4a = 8$, $a = 2$

Focus : $S \equiv (0,a) \equiv (0,2)$

Equation of directrix : $y + a = 0 \therefore y + 2 = 0$

Length of latus recturm $= 4a = 8$ units

Ends of latus rectum : $L \equiv (2a , a) \equiv (4,2)$

$L' \equiv (-2a,a) \equiv (-4,2)$

(5) $x^2 + 12y = 0$

$x^2 = -12y$

SOLUTION :

Equation of parabola : $x^2 = -12y$

On comparing with : $x^2 = -4ay$, $4a = 12$, $a = 3$

Focus : $S \equiv (0,-a) \equiv (0,-3)$

Equation of directrix : $y - a = 0 \therefore y - 3 = 0$

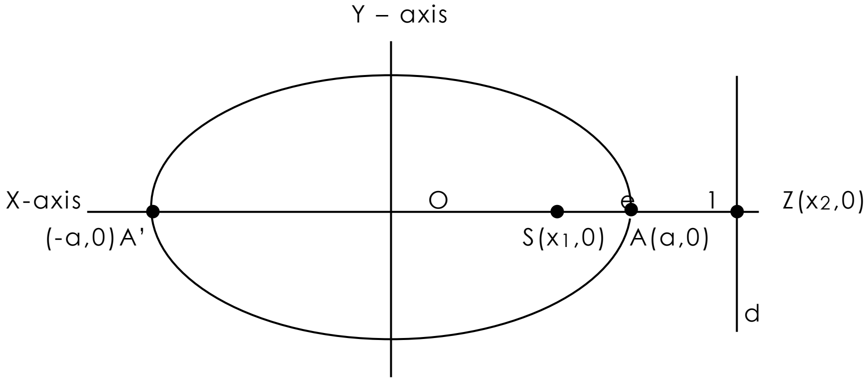
Length of latus recturm $= 4a = 12$ units

Ends of latus rectum : $L \equiv (2a , -a) \equiv (6,-3)$

$L' \equiv (-2a,-a) \equiv (-6,-3$

ELLIPSE

Derive Standard form Of Ellipse : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



S : focus
Directrix d cuts x axis at Z
e : eccentricity

Let A(a,0) & A'(-a,0)
Let Y axis bisect AA'

Part I : Finding focus & directrix

By focus – directrix property : $\frac{AS}{AZ} = \frac{A'S}{A'Z} = e$

Hence ;

A divides SZ internally in the ratio e:1

$\therefore\ a = \frac{ex_2 + x_1}{e + 1}$

$ae + a = ex_2 + x_1$ .... (1)

A' divides SZ externally in the ratio e:1

$\therefore\ -a = \frac{ex_2 - x_1}{e - 1}$

$-ae + a = ex_2 - x_1$ .... (2)

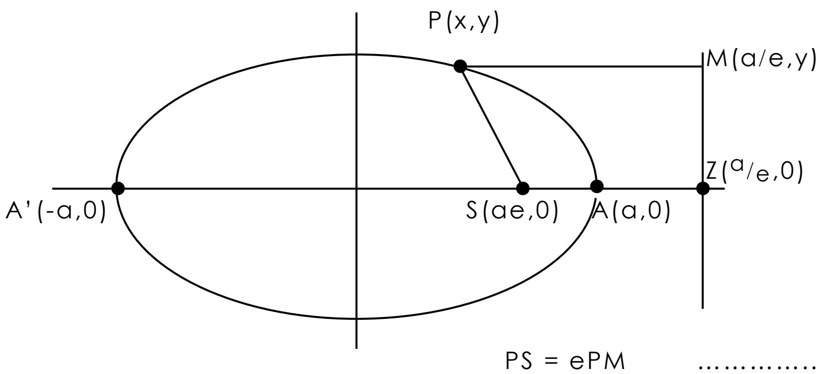
Solving (1) & (2)

$$ae + a = ex_2 + x_1$$
$$-ae + a = ex_2 - x_1$$

(1) + (2) $\quad 2a = 2ex_2 \quad \therefore\ x_2 = {}^{a}/_{e}$ $\quad$ Hence $Z \equiv ({}^{a}/_{e}, 0)$

subs $x_2 = a/e$ in (1) $\quad ae + a = a + x_1 \quad \therefore\ x_1 = ae$ $\quad$ Hence $S \equiv (ae,0)$

Part II : Generating Equation of the Ellipse



Let P(x,y) be any point on the ellipse
Draw PM ⊥ directrix ,$M({}^{a}/_{e},y)$

$PS = ePM$ .... Using focus-directrix property

$$PS^2 = e^2.PM^2$$

$$(x - ae)^2 + y^2 = e^2[\ (x - {}^{a}/_{e})^2 + (y - y)^2\ ]$$

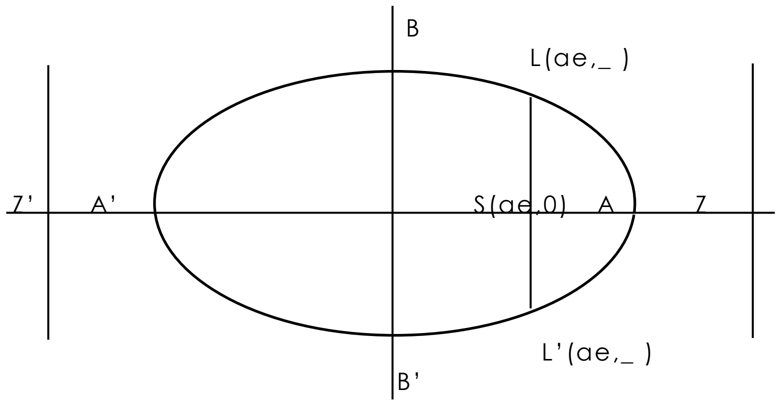
$$x^2 - 2aex + a^2e^2 + y^2 = e^2(x^2 - 2x{}^{a}/_{e} + a^2/e^2)$$

$$x^2 - 2aex + a^2e^2 + y^2 = e^2x^2 - 2aex + a^2$$

$$x^2(1-e^2) + y^2 = a^2(1 - e^2)$$

Dividing thro' by $a^2(1-e^2)$ $\quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad$ where $b^2 = a^2(1-e^2)$

ENDS AND LENGTH OF THE LATUS RECTUM AT S



Equation of ellipse : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

subs x = ae in the above equation

$$\frac{a^2e^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$y^2 = b^2(1 - e^2)$$

$$= \frac{b^2.b^2}{a^2}$$

$$= \frac{b^4}{a^2}$$

$$y = \pm b^2/a$$

$b^2 = a^2(1 - e^2)$
$\therefore\ 1 - e^2 = b^2/a^2$

hence : $L(ae,b^2/a)$; $L'(ae,-b^2/a)$

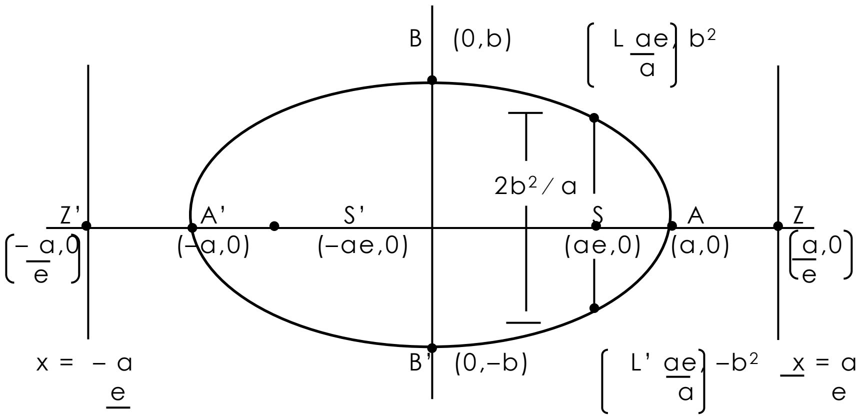
Now length of the latus-rectum : $LL' = \sqrt{(ae - ae)^2 + (b^2/a + b^2/a)^2}$

$$= 2b^2/a$$

a > b

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b^2 = a^2(1 - e^2)$$



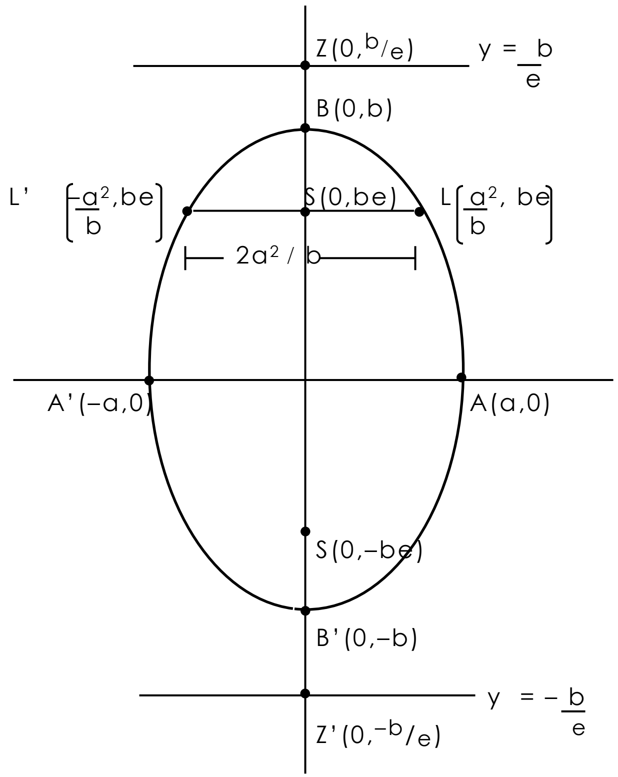
2ae — dist between foci

$\frac{2a}{e}$ — distance between directrices

b > a

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$a^2 = b^2(1 - e^2)$$



distance between foci = 2be

distance between directrices = $\frac{2b}{e}$

Q. SET - 1

Find equation of ellipse referred to its principal axes

1) whose foci (± 4,0) and eccentricity = 1/3 ans : $\frac{x^2}{144} + \frac{y^2}{128} = 1$

2) whose foci (± 12,0) and eccentricity = 12/13 ans : $\frac{x^2}{169} + \frac{y^2}{25} = 1$

3) whose foci (± 3,0) and eccentricity = 2/3 ans : $\frac{4x^2}{81} + \frac{4y^2}{45} = 1$

4) whose foci (± 3,0) and cutting x-axis at (± 5,0) ans : $\frac{x^2}{25} + \frac{y^2}{16} = 1$

5) whose foci (0,±2) and cutting y-axis at (0,±3) ans : $\frac{x^2}{5} + \frac{y^2}{9} = 1$

6) distance between foci is 6 and e = 3/5 ans : $\frac{x^2}{25} + \frac{y^2}{16} = 1$

7) distance between directrices is 10 & eccentricity = 1/√5 ans : $\frac{x^2}{5} + \frac{y^2}{4} = 1$

8) distance between foci = 8 and major axis = 10 ans : $\frac{x^2}{25} + \frac{y^2}{9} = 1$

9) whose latus rectum = 39/4 and eccentricity = 5/8 ans : $\frac{x^2}{64} + \frac{y^2}{39} = 1$

10) whose latus rectum = 5/2 and eccentricity = 1/2 ans : $\frac{9x^2}{25} + \frac{12y^2}{25} = 1$

11) whose length of latus rectum is 50/13 and minor axis is 10 ans : $\frac{x^2}{169} + \frac{y^2}{25} = 1$

12) distance between directrices is 5√5 and distance between foci is 4√5

ans : $\frac{x^2}{25} + \frac{y^2}{5} = 1$

13) eccentricity = √3/2 and passing through (6,−4) ans : $\frac{x^2}{100} + \frac{y^2}{25} = 1$

14) eccentricity = $^2/_3$ and passing through $(2, ^5/_3)$ ans : $\frac{x^2}{5} + \frac{y^2}{9} = 1$

15) passing through (–3 , 1) and (2 , –2) ans : $\frac{3x^2}{32} + \frac{5y^2}{32} = 1$

16) passing through (1 , 4) and (–6 , –1) ans : $\frac{3x^2}{115} + \frac{7y^2}{115} = 1$

17) distance between foci is 8 and passing through $(\sqrt{15},-1)$ ans : $\frac{x^2}{20} + \frac{y^2}{4} = 1$

18) distance between directrices is 10 and passing through $(-\sqrt{5},2)$
ans : $\frac{x^2}{15} + \frac{y^2}{6} = 1$

Q. SET - 2

Find eccentricity of ellipse given

01) length of latus rectum is half of its major axis ans : $^1/_{\sqrt{2}}$

02) length of latus rectum is $^1/_3$ minor axis ans : $^{2\sqrt{2}}/_3$

03) length of minor axis is equal to distance between foci ans : $^1/_{\sqrt{2}}$

04) length of latus rectum is semi minor axis ans : $^{\sqrt{3}}/_2$

05) distance between directrices is three times the distance between foci
ans : $^1/_{\sqrt{3}}$

Q. SET - 3

Find eccentricity , coordinates of foci , equation of directrices , length of major and minor axes and length of latus rectum

01) $16x^2 + 25y^2 = 400$

ans : $e = ^3/_5$; foci $(\pm 3,0)$; $x = \pm ^{25}/_3$; 10 ; 8 ; $^{32}/_5$

02) $4x^2 + 5y^2 = 20$

ans : $e = ^1/_{\sqrt{5}}$; foci $(\pm 1,0)$; $x = \pm 5$; $2\sqrt{5}$; 4 ; $^8/_{\sqrt{5}}$

03) $3x^2 + 4y^2 = 1$

ans : $e = \frac{1}{2}$; foci $(\pm \frac{1}{2\sqrt{3}}, 0)$; $x = \pm \frac{2}{\sqrt{3}}$; $\frac{2}{\sqrt{3}}$; 1 ; $\frac{\sqrt{3}}{2}$

04) $9x^2 + 4y^2 = 36$

ans : $e = \frac{\sqrt{5}}{3}$; foci $(0, \pm\sqrt{5})$; $y = \pm \frac{9}{\sqrt{5}}$; 6 ; 4 ; $\frac{8}{3}$

05) $9x^2 + 5y^2 = 45$

ans : $e = \frac{2}{3}$; foci $(0, \pm 2)$; $y = \pm \frac{9}{2}$; 6 ; $2\sqrt{5}$; $\frac{10}{3}$

SOLUTION TO - Q SET 1

Find equation of ellipse referred to its principal axes

01.

whose foci (± 4 , 0) and eccentricity = $^1/_3$

Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

foci ≡ (±ae , 0) ≡ (±4 , 0)

$\therefore$ ae = 4 …… (1)

e = $\frac{1}{3}$ ….. given

subs in (1) : $a \cdot \frac{1}{3} = 4$

$a = 12$

Now ; $b^2 = a^2(1 - e^2)$

$b^2 = 144 \left(1 - \frac{1}{9}\right)$

$b^2 = 144 \times \frac{8}{9}$

$b^2 = 16 \times 8 = 128$

Hence ,
equation of the ellipse : $\frac{x^2}{144} + \frac{y^2}{128} = 1$

02.

whose foci (± 12 , 0) and eccentricity = $^{12}/_{13}$

Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

foci ≡ (±ae , 0) ≡ (±12 , 0)

$\therefore$ ae = 12 …… (1)

e = $\frac{12}{13}$ ….. given

subs in (1) : $a \cdot \frac{12}{13} = 12$

$a = 13$

Now ; $b^2 = a^2(1 - e^2)$

$b^2 = 169 \left(1 - \frac{144}{169}\right)$

$b^2 = 169 \times \frac{25}{169}$

$b^2 = 25$

Hence ,
equation of the ellipse : $\frac{x^2}{169} + \frac{y^2}{25} = 1$

03.

whose foci (± 3 , 0) and eccentricity = $^2/_3$

Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

foci ≡ (±ae , 0) ≡ (±3 , 0)

$\therefore$ ae = 3 …… (1)

e = $\frac{2}{3}$ ….. given

subs in (1) : $a \cdot \frac{2}{3} = 3$

$a = {}^9/_2$

Now ; $b^2 = a^2(1 - e^2)$

$b^2 = \frac{81}{4} \left(1 - \frac{4}{9}\right)$

$b^2 = \frac{81}{4} \times \frac{5}{9}$

$b^2 = \frac{45}{4}$

Hence ,
equation of the ellipse : $\frac{x^2}{\frac{81}{4}} + \frac{y^2}{\frac{45}{4}} = 1$

: $\frac{4x^2}{81} + \frac{4y^2}{45} = 1$

04.

whose foci (± 3 , 0) and cutting x – axis at (±3,0)

Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

foci ≡ (±ae , 0) ≡ (±3 , 0)

$\therefore$ ae = 3 …… (1)

cutting x – axis at (± a , 0) ≡ (± 5, 0)

$\therefore$ a = 5

subs in (1) : 5.e = 3

e = $^3/_5$

Now ; $b^2 = a^2(1 - e^2)$

$b^2 = 25 \left(1 - \frac{9}{25}\right)$

$b^2 = 25 \times \frac{16}{25}$

$b^2 = 16$

Hence ,
equation of the ellipse : $\frac{x^2}{25} + \frac{y^2}{16} = 1$

05.

whose foci (0 , ± 2) and cutting y – axis at (0, ± 3)

Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

foci ≡ (0 , ±be) ≡ (0 , ± 2)

$\therefore$ be = 2 …… (1)

cutting y – axis at (0 , ±b) ≡ (0 , ±3)

$\therefore$ b = 3

subs in (1) : 3.e = 2

e = $^2/_3$

Now ; $a^2 = b^2(1 - e^2)$

$a^2 = 9 \left(1 - \frac{4}{9}\right)$

$a^2 = 9 \times \frac{5}{9}$

$a^2 = 5$

Hence ,
equation of the ellipse : $\frac{x^2}{5} + \frac{y^2}{9} = 1$

06.

whose distance between foci is 6 & e = $^3/_5$

Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

distance between foci = 6

$\therefore$ 2ae = 3

ae = 3 …… (1)

e = $^3/_5$ ……. Given

subs in (1) : $a.\frac{3}{5} = 3$

a = 5

Now ; $b^2 = a^2(1 - e^2)$

$b^2 = 25 \left(1 - \frac{9}{25}\right)$

$b^2 = 25 \times \frac{16}{25}$

$b^2 = 16$

Hence ,
equation of the ellipse : $\frac{x^2}{25} + \frac{y^2}{16} = 1$

07.

distance between directrices is 10 & e = $1/\sqrt{5}$

Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

distance between directrices = 10

$$\therefore \frac{2a}{e} = 10$$

$$\frac{a}{e} = 5 \quad \ldots\ldots \quad (1)$$

$$e = 1/\sqrt{5} \quad \ldots\ldots\text{. Given}$$

subs in (1) :

$$\frac{a}{\frac{1}{\sqrt{5}}} = 5$$

$$\sqrt{5}a = 5$$

$$a = \sqrt{5}$$

Now ; $b^2 = a^2(1 - e^2)$

$$b^2 = 5\left(1 - \frac{1}{5}\right)$$

$$b^2 = 5 \times \frac{4}{5}$$

$$b^2 = 4$$

Hence ,
equation of the ellipse : $\frac{x^2}{5} + \frac{y^2}{4} = 1$

08.

whose distance between foci is 8 and major axis is 10

Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

distance between foci = 8

$$\therefore 2ae = 8$$

$$ae = 4 \quad \ldots\ldots \quad (1)$$

major axis = 10

$$2a = 10$$

$$\therefore a = 5$$

subs in (1) : $5.e = 4$

$$e = 4/5$$

Now ; $b^2 = a^2(1 - e^2)$

$$b^2 = 25\left(1 - \frac{16}{25}\right)$$

$$b^2 = 25 \times \frac{9}{25}$$

$$b^2 = 9$$

Hence ,
equation of the ellipse : $\frac{x^2}{25} + \frac{y^2}{9} = 1$

09.

whose latus – rectum = $39/4$ and e = $5/8$

Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

latus – rectum = $\frac{39}{4}$

$$\therefore \frac{2b^2}{a} = \frac{39}{4}$$

$$b^2 = \frac{39a}{8} \quad \ldots\ldots \quad (1)$$

$$e = 5/8 \quad \ldots\ldots \quad \text{given}$$

Now ; $b^2 = a^2(1 - e^2)$

$$\frac{39a}{8} = a^2\left(1 - \frac{25}{64}\right)$$

$$\frac{39a}{8} = a^2\,\frac{39}{64}$$

$$a = 8$$

subs on (1) : $b^2 = \frac{39(8)}{8}$

$b^2 = 39$

Hence ,
equation of the ellipse : $\frac{x^2}{64} + \frac{y^2}{39} = 1$

10.

whose latus – rectum $= {}^{5}/_{2}$ and $e = {}^{1}/_{2}$

Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

latus – rectum $= \frac{5}{2}$

$\therefore \frac{2b^2}{a} = \frac{5}{2}$

$b^2 = \frac{5a}{4}$ …… (1)

$e = {}^{1}/_{2}$ ….. given

Now ; $b^2 = a^2(1 - e^2)$

$\frac{5a}{4} = a^2\left(1 - \frac{1}{4}\right)$

$\frac{5a}{4} = a^2 \frac{3}{4}$

$5a = 3a^2$

$a = {}^{5}/_{3}$

subs on (1) : $b^2 = \frac{5}{4}\,\frac{5}{3}$

$b^2 = \frac{25}{12}$

Hence ,
equation of the ellipse : $\frac{x^2}{\frac{25}{9}} + \frac{y^2}{\frac{25}{12}} = 1$

$\frac{9x^2}{25} + \frac{12y^2}{25} = 1$

11.

latus – rectum $= {}^{50}/_{13}$ and minor axis = 10

Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

latus – rectum $= \frac{50}{13}$

$\therefore \frac{2b^2}{a} = \frac{50}{13}$

$b^2 = \frac{25a}{13}$ …… (1)

minor axis = 10

$2b = 10$

$b = 5$

subs in (1)

$25 = \frac{25a}{13}$ $\therefore a = 13$

Hence ,
equation of the ellipse : $\frac{x^2}{169} + \frac{y^2}{25} = 1$

12.

whose distance between directrices is $5\sqrt{5}$ & distance between foci $= 4\sqrt{5}$

Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

distance between directrices $= 5\sqrt{5}$

$$\therefore \frac{2a}{e} = 5\sqrt{5}$$

$$\frac{a}{e} = \frac{5\sqrt{5}}{2} \quad \ldots\ldots \quad (1)$$

distance between foci $= 4\sqrt{5}$

$$2ae = 4\sqrt{5}$$

$$ae = 2\sqrt{5} \quad \ldots\ldots \quad (2)$$

(1) x (2)

$$\frac{a}{e} \times ae = \frac{5\sqrt{5}}{2} \times 2\sqrt{5}$$

$$a^2 = 25$$

$$a = 5$$

subs in (2)

$$5e = 2\sqrt{5}$$

$$e = \frac{2\sqrt{5}}{5}$$

$$e = \frac{2}{\sqrt{5}}$$

Now ; $b^2 = a^2(1 - e^2)$

$$b^2 = 25\left(1 - \frac{4}{5}\right)$$

$$b^2 = 25 \times \frac{1}{5}$$

$$b^2 = 5$$

Hence ,
equation of the ellipse : $\frac{x^2}{25} + \frac{y^2}{5} = 1$

13.

eccentricity $= \sqrt{3}/2$ and passing through $(6,-4)$

Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$e = \sqrt{3}/2$

Now ; $b^2 = a^2(1 - e^2)$

$$b^2 = a^2\left(1 - \frac{3}{4}\right)$$

$$b^2 = \frac{a^2}{4}$$

$$a^2 = 4b^2 \quad \ldots.. \quad (1)$$

Since ellipse is passing through $(6,-4)$, it must satisfy the equation of the ellipse

$$\frac{36}{a^2} + \frac{16}{b^2} = 1 \quad \ldots\ldots\ldots \quad (2)$$

Solving (1) & (2)

$$\frac{36}{4b^2} + \frac{16}{b^2} = 1$$

$$\frac{9}{b^2} + \frac{16}{b^2} = 1$$

$$b^2 = 25$$

subs in (1) $a^2 = 4(25) = 100$

Hence ,
equation of the ellipse : $\frac{x^2}{100} + \frac{y^2}{25} = 1$

14.

eccentricity = $^2/_3$ and passing through $(2, {}^5/_3)$

Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$e = {}^2/_3$

Now ; $b^2 = a^2(1 - e^2)$

$$b^2 = a^2\left(1 - \frac{4}{9}\right)$$

$$b^2 = \frac{5a^2}{9} \quad \ldots\ldots \quad (1)$$

Since ellipse is passing through $(2, {}^5/_3)$, it must satisfy the equation of the ellipse

$$\frac{4}{a^2} + \frac{25}{9b^2} = 1 \quad \ldots\ldots \quad (2)$$

Solving (1) & (2)

$$\frac{4}{a^2} + \frac{25}{9\,\frac{5a^2}{9}} = 1$$

$$\frac{4}{a^2} + \frac{5}{a^2} = 1$$

$$a^2 = 9$$

subs in (1) $b^2 = \frac{5(9)}{9} = 5$

Hence ,

equation of the ellipse : $\frac{x^2}{9} + \frac{y^2}{5} = 1$

15.

passing through $(-3 , 1)$ and $(2 , -2)$

Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Since ellipse is passing thro' $(-3 , 1)$ & $(2 , -2)$, they must satisfy the equation of the ellipse

$$\frac{9}{a^2} + \frac{1}{b^2} = 1 \quad \ldots\ldots\ (1)$$

$$\frac{4}{a^2} + \frac{4}{b^2} = 1 \quad \ldots\ldots\ (2)$$

Solving (1) and (2)

$$\frac{36}{a^2} + \frac{4}{b^2} = 4$$

$$-\ \frac{4}{a^2} - \frac{4}{b^2} - 1$$

$$\frac{32}{a^2} = 3$$

$$a^2 = \frac{32}{3}$$

subs in (1)

$$\frac{9}{\frac{32}{3}} + \frac{1}{b^2} = 1 \quad \ldots\ldots\ (1)$$

$$\frac{27}{32} + \frac{1}{b^2} = 1$$

$$\frac{1}{b^2} = 1 - \frac{27}{32}$$

$$\frac{1}{b^2} = \frac{5}{32}$$

$$b^2 = \frac{32}{5}$$

Hence ,

equation of the ellipse : $\frac{3x^2}{32} + \frac{5y^2}{32} = 1$

16.

passing through (1 , 4) and (−6 , −1)

Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Since ellipse is passing thro' (1 , 4) & (−6 , −1), they must satisfy the equation of the ellipse

$$\frac{1}{a^2} + \frac{16}{b^2} = 1 \quad .... (1)$$

$$\frac{36}{a^2} + \frac{1}{b^2} = 1 \quad .... (2)$$

Solving (1) and (2)

$$\frac{36}{a^2} + \frac{576}{b^2} = 36$$

$$\underline{-\frac{36}{a^2} - \frac{1}{b^2} = -1}$$

$$\frac{575}{b^2} = 35$$

$$b^2 = \frac{575}{35} = \frac{115}{7}$$

subs in (1)

$$\frac{1}{a^2} + \frac{16}{\frac{115}{7}} = 1 \quad .... (1)$$

$$\frac{1}{a^2} + \frac{112}{115} = 1$$

$$\frac{1}{a^2} = 1 - \frac{112}{115}$$

$$\frac{1}{a^2} = \frac{3}{115}$$

$$a^2 = \frac{115}{3}$$

Hence ,
equation of the ellipse : $\frac{3x^2}{115} + \frac{7y^2}{115} = 1$

17.

distance between foci is 8 and passing through (√15,−1)

Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

distance between foci $= 8$

$$2ae = 8$$

$$e = \frac{4}{a} \quad .... (1)$$

Since ellipse is passing thro' (√15,−1), it must satisfy the equation of the ellipse

$$\frac{15}{a^2} + \frac{1}{b^2} = 1 \quad .... \quad (2)$$

Also $\quad b^2 = a^2(1 - e^2)$

$$b^2 = a^2\left(1 - \frac{16}{a^2}\right) \quad \text{from (1)}$$

$$b^2 = a^2\left(\frac{a^2 - 16}{a^2}\right)$$

$$b^2 = a^2 - 16 \quad .... \quad (3)$$

Solving (2) & (3)

$$\frac{15}{a^2} + \frac{1}{a^2 - 16} = 1$$

$$15a^2 - 240 + a^2 = a^4 - 16a^2$$

$$16a^2 - 240 = a^4 - 16a^2$$

$$a^4 - 32a^2 - 240 = 0$$

$$(a^2 - 12)(a^2 - 20) = 0$$

$a^2 = 12 \quad ; \quad a^2 = 20$

subs in (3)

$b^2 = -4 \quad ; \quad b^2 = 4$

DISCARD

Hence ,
equation of the ellipse : $\frac{x^2}{20} + \frac{y^2}{4} = 1$

18. distance between directrices is 10 and passing through $(-\sqrt{5}, 2)$

Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

distance between directrices = 10

$$\frac{2a}{e} = 10$$

$$e = \frac{a}{5} \quad \ldots\ldots (1)$$

Since ellipse is passing thro' $(-\sqrt{5}, 2)$, it must satisfy the equation of the ellipse

$$\frac{5}{a^2} + \frac{4}{b^2} = 1 \quad \ldots\ldots \quad (2)$$

Also $\quad b^2 = a^2(1 - e^2)$

$$b^2 = a^2\left(1 - \frac{a^2}{25}\right)$$

$$b^2 = a^2\left(\frac{25 - a^2}{25}\right) \ldots\ldots (3)$$

Solving (2) & (3)

$$\frac{5}{a^2} + \frac{4}{a^2\left(\frac{25 - a^2}{25}\right)} = 1$$

$$\frac{5}{a^2} + \frac{100}{a^2\,(25 - a^2)} = 1$$

$$\frac{5(25 - a^2) + 100}{a^2(25 - a^2)} = 1$$

$$125 - 5a^2 + 100 = 25a^2 - a^4$$

$$a^4 - 30a^2 + 225 = 0$$

$$(a^2 - 15)(a^2 - 15) = 0$$

$$a^2 = 15$$

subs in (3)

$$b^2 = 15\left(\frac{25 - 15}{25}\right) = 6$$

SOLUTION TO - Q SET 2

01.
length of latus rectum is half of its major axis

for the ellipse : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Given :
length of latus rectum is half of its major axis

$$\frac{2b^2}{a} = \frac{1}{2}(2a)$$

$$2b^2 = a^2 \quad \ldots\ldots \quad (1)$$

Also ; $\quad b^2 = a^2(1 - e^2)$

$$b^2 = 2b^2\,(1 - e^2) \quad \ldots\ldots \quad \text{From (1)}$$

$$1 = 2 - 2e^2$$

$$2e^2 = 1$$

$$e^2 = 1/2$$

$$e = 1/\sqrt{2} \ .$$

02.
length of latus rectum is $1/3$ minor axis

for the ellipse : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Given :
length of latus rectum is $1/3$ minor axis

$$\frac{2b^2}{a} = \frac{1}{3}(2b)$$

$$b = \frac{a}{3} \quad \ldots\ldots \quad (1)$$

Also ; $\quad b^2 = a^2(1 - e^2)$

$$\frac{a^2}{9} = a^2\,(1 - e^2) \ \ldots\ldots\text{From (1)}$$

$$\frac{1}{9} = 1 - e^2$$

$$e^2 = 8/9$$

$$e = 2\sqrt{2}/3$$

03.

length of minor axis is equal to distance between foci

for the ellipse : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Given :

Length of minor axis is equal to distance between foci

$2b = 2ae$

$b = ae$ ...... (1)

Also ; $b^2 = a^2(1 - e^2)$

$a^2 e^2 = a^2 (1 - e^2)$ .....From (1)

$e^2 = 1 - e^2$

$2e^2 = 1$

$e^2 = {}^1/_2$

$e = {}^1/_{\sqrt{2}}$.

04.

length of latus rectum is semi minor axis

for the ellipse : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Given :

length of latus rectum is semi minor axis

$\frac{2b^2}{a} = b$

$2b = a$ ..... (1)

Also ; $b^2 = a^2(1 - e^2)$

$b^2 = 4b^2 (1 - e^2)$ ..... From (1)

$\frac{1}{4} = 1 - e^2$

$e^2 = \frac{3}{4}$

$e = {}^{\sqrt{3}}/_2$.

05.

distance between directrices is three times the distance between foci

for the ellipse : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Given :

distance between directrices is three times the distance between foci

$\frac{2a}{e} = 3(2ae)$

$\frac{1}{e} = 3e$

$e^2 = {}^1/_3$

$e = {}^1/_{\sqrt{3}}$.

SOLUTION TO - Q SET 3

Find eccentricity , coordinates of foci , equation of directrices , length of major and minor axes and length of latus rectum

01. $16x^2 + 25y^2 = 400$

$\frac{16x^2}{400} + \frac{25y^2}{400} = 1$

$\frac{x^2}{25} + \frac{y^2}{16} = 1$

$a^2 = 25 \quad \therefore a = 5$

$b^2 = 16 \quad \therefore b = 4 \qquad a > b$

Eccentricity

$b^2 = a^2(1 - e^2)$

$16 = 25(1 - e^2)$

$\frac{16}{25} = 1 - e^2$

$e^2 = 1 - \frac{16}{25}$

$e^2 = \frac{9}{25}$

✓ $e = {}^3/_5$

$ae = 5 \times {}^3/_5 = 3$

$\frac{a}{e} = \frac{5}{{}^3/_5} = \frac{25}{3}$

✓ foci $\equiv (\pm ae , 0) \equiv (\pm 3,0)$

✓ eq. of directrices : $x = \pm {}^a/_e$

$x = \pm {}^{25}/_3$

✓ length of major axis $= 2a = 10$

✓ length of minor axis $= 2b = 8$

✓ length of latus rectum $= \frac{2b^2}{a} = \frac{32}{5}$

02. $4x^2 + 5y^2 = 20$

$\frac{4x^2}{20} + \frac{5y^2}{20} = 1$

$\frac{x^2}{5} + \frac{y^2}{4} = 1$

$a^2 = 5 \quad \therefore a = \sqrt{5}$

$b^2 = 4 \quad \therefore b = 2 \qquad a > b$

Eccentricity

$b^2 = a^2(1 - e^2)$

$4 = 5(1 - e^2)$

$\frac{4}{5} = 1 - e^2$

$e^2 = 1 - \frac{4}{5}$

$e^2 = \frac{1}{5}$

✓ $e = {}^1/_{\sqrt{5}}$

$ae = \sqrt{5} \times {}^1/_{\sqrt{5}} = 1$

$\frac{a}{e} = \frac{\sqrt{5}}{{}^1/_{\sqrt{5}}} = 5$

✓ foci $\equiv (\pm ae , 0) \equiv (\pm 1,0)$

✓ eq. of directrices : $x = \pm {}^a/_e$

$x = \pm 5$

✓ length of major axis $= 2a = 2\sqrt{5}$

✓ length of minor axis $= 2b = 4$

✓ length of latus rectum $= \frac{2b^2}{a} = \frac{8}{\sqrt{5}}$

03. $3x^2 + 4y^2 = 1$

$$\frac{x^2}{^1/_3} + \frac{y^2}{^1/_4} = 1$$

$a^2 = {}^1/_3 \quad \therefore a = {}^1/_{\sqrt{3}}$

$b^2 = {}^1/_4 \quad \therefore b = {}^1/_2 \quad a > b$

Eccentricity

$b^2 = a^2(1 - e^2)$

$\frac{1}{4} = \frac{1}{3}(1 - e^2)$

$\frac{3}{4} = 1 - e^2$

$e^2 = 1 - \frac{3}{4}$

$e^2 = \frac{1}{4}$

✓ $e = {}^1/_2$

$ae = {}^1/_2 \times {}^1/_{\sqrt{3}} = {}^1/_{2\sqrt{3}}$

$\frac{a}{e} = \frac{^1/_{\sqrt{3}}}{^1/_2} = {}^2/_{\sqrt{3}}$

✓ foci $\equiv (\pm ae\ ,\ 0) \equiv (\pm {}^1/_{2\sqrt{3}}\ ,\ 0)$

✓ eq. of directrices : $x = \pm\, {}^a/_e$

$x = \pm\, {}^2/_{\sqrt{3}}$

✓ length of major axis $= 2a = {}^2/_{\sqrt{3}}$

✓ length of minor axis $= 2b = 1$

✓ length of latus rectum $= \frac{2b^2}{a} = \frac{2(^1/_4)}{^1/_{\sqrt{3}}}$

$= \frac{^1/_2}{^1/_{\sqrt{3}}}$

$= {}^{\sqrt{3}}/_2$

04. $9x^2 + 4y^2 = 36$

$$\frac{9x^2}{36} + \frac{4y^2}{36} = 1$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

$a^2 = 4 \quad \therefore a = 2$

$b^2 = 9 \quad \therefore b = 3 \quad b > a$

Eccentricity

$a^2 = b^2(1 - e^2)$

$4 = 9(1 - e^2)$

$\frac{4}{9} = 1 - e^2$

$e^2 = 1 - \frac{4}{9}$

$e^2 = \frac{5}{9}$

✓ $e = {}^{\sqrt{5}}/_3$

$be = 3 \times {}^{\sqrt{5}}/_3 = \sqrt{5}$

$\frac{b}{e} = \frac{3}{^{\sqrt{5}}/_3} = \frac{9}{\sqrt{5}}$

✓ foci $\equiv (0\ ,\ \pm be)) \equiv (0\ ,\ \pm\sqrt{5})$

✓ eq. of directrices : $y = \pm\, {}^b/_e$

$y = \pm\, {}^9/_{\sqrt{5}}$

✓ length of major axis $= 2b = 6$

✓ length of minor axis $= 2a = 4$

✓ length of latus rectum $= \frac{2a^2}{b} = \frac{8}{3}$

05. $9x^2 + 4y^2 = 36$

$$\frac{9x^2}{36} + \frac{4y^2}{36} = 1$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

$a^2 = 4 \quad \therefore a = 2$

$b^2 = 9 \quad \therefore b = 3 \qquad b > a$

Eccentricity

$a^2 = b^2(1 - e^2)$

$4 = 9(1 - e^2)$

$$\frac{4}{9} = 1 - e^2$$

$$e^2 = 1 - \frac{4}{9}$$

$$e^2 = \frac{5}{9}$$

✓ $e = {}^{\sqrt{5}}/_{3}$

$be = 3 \times {}^{\sqrt{5}}/_{3} = \sqrt{5}$

$$\frac{b}{e} = \frac{3}{{}^{\sqrt{5}}/_{3}} = \frac{9}{\sqrt{5}}$$

✓ foci $\equiv (0, \pm be)) \equiv (0, \pm\sqrt{5})$

✓ eq. of directrices : $y = \pm\, {}^{b}/_{e}$

$y = \pm\, {}^{9}/_{\sqrt{5}}$

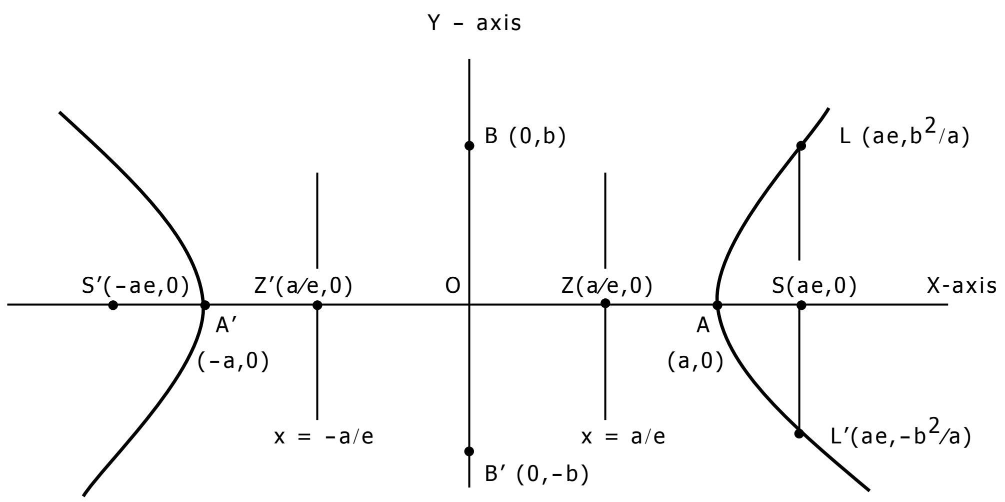
✓ length of major axis $= 2b = 6$

✓ length of minor axis $= 2a = 4$

✓ length of latus rectum $= \dfrac{2a^2}{b} = \dfrac{8}{3}$

HYPERBOLA

$$\frac{\mathbf{x^2}}{\mathbf{a^2}} - \frac{\mathbf{y^2}}{\mathbf{b^2}} = \mathbf{1} \quad ; \quad b^2 = a^2(e^2 - 1)$$



length of the focal/transverse axis = 2a

Length of the conjugate axis = 2b

foci : S $\equiv$ ($\pm$ ae,0) ; distance between foci = 2ae

directrices : x = $\pm$ a ; distance between directrices = $\frac{2a}{e}$

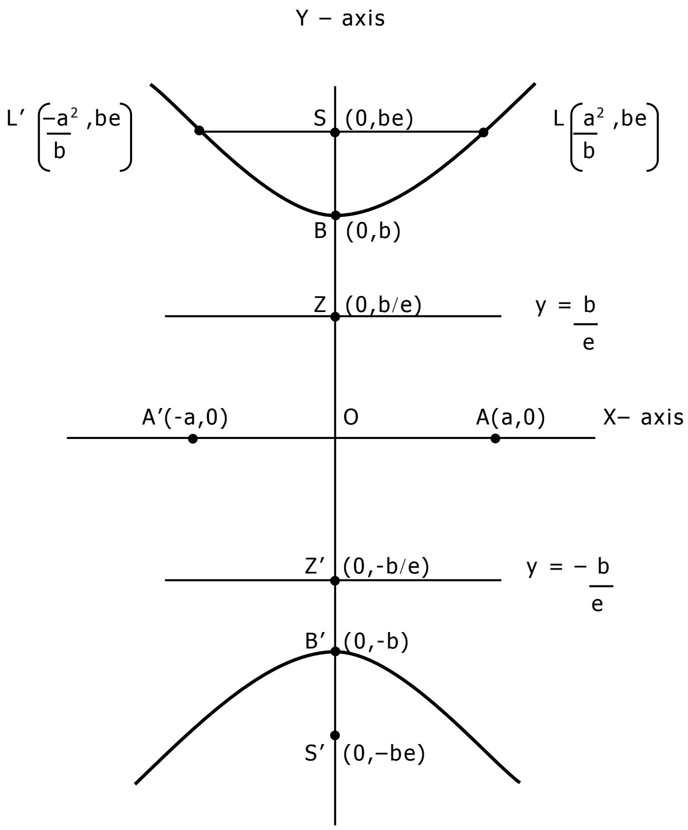
Latus – Rectum :

Length = $\frac{2b^2}{a}$

Endpoints : $L \equiv \left(ae , \frac{b^2}{a}\right)$; $L' \equiv \left(ae, -\frac{b^2}{a}\right)$ (at S)

Equation : x = $\pm$ ae

CONJUGATE HYPERBOLA



$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

$$a^2 = b^2(e^2 - 1)$$

Q1. find equation of hyperbola referred to principal axes

1. length of the transverse axis & length of latus rectum are 18 & 14 respectively

   ans : $\frac{x^2}{81} - \frac{y^2}{63} = 1$

2. eccentricity = 3/2 & distance between foci = 6

   ans : $\frac{x^2}{4} - \frac{y^2}{5} = 1$

3. eccentricity = 3/2 & distance between foci = 12

   ans : $\frac{x^2}{16} - \frac{y^2}{20} = 1$

4. eccentricity = √2 & distance between foci = 8√2

   ans : $\frac{x^2}{16} - \frac{y^2}{16} = 1$

5. eccentricity = 5/3 & distance between directrices = 18/5

   ans : $\frac{x^2}{16} - \frac{y^2}{16} = 1$

6. foci = (±8,0) & vertex at (6,0)

   ans : $\frac{x^2}{36} - \frac{y^2}{28} = 1$

7. eccentricity = √2 & passing through (–5,3)

   ans : $\frac{x^2}{16} - \frac{y^2}{16} = 1$

8. conjugate axis = 8 & distance between foci = 10

ans : $\frac{x^2}{9} - \frac{y^2}{16} = 1$

9. conjugate axis = 5 & distance between foci = 13

ans : $\frac{x^2}{36} - \frac{y^2}{25} = 1$

10. transverse axis = $2\sqrt{2}$ & passing thro' (2,–3)

ans : $\frac{x^2}{2} - \frac{y^2}{9} = 1$

11. transverse axis = 7 & passing thro' (5,–2)

ans : $\frac{4x^2}{49} - \frac{51y^2}{196} = 1$

12. length of latus rectum is equal to square of the length of conjugate axis , passes through (–2,3)

ans : $4x^2 - \frac{5y^2}{3} = 1$

13. passing through (2,1) & (4,3)

ans : $\frac{2x^2}{5} - \frac{3y^2}{5} = 1$

14. passing through (6, –1) & $(-8, 2\sqrt{2})$

ans : $\frac{x^2}{32} - \frac{y^2}{8} = 1$

15. foci of hyperbola coincides with the foci of ellipse $9x^2 + 16y^2 = 144$; $e = \sqrt{2}$

ans : $\frac{2x^2}{7} - \frac{2y^2}{7} = 1$

Q2.

01. for : $\frac{x^2}{100} - \frac{y^2}{25} = 1$

Prove that :

a) $e = \sqrt{5}/2$ b) SA.S'A = 25 where S, S' are foci , A is vertex

02. for hyperbola : $x^2 - 2y^2 = 1$, find

a) e b) |SA – S'A| c) SA.S'A

where S and S' are the foci and A is the vertex

ans : $\sqrt{3}/\sqrt{2}$, 2 , 1/2

03. if e_1 and e_2 are the eccentricities of hyperbolas

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ & $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$

then prove that : $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$

Q3.

find length of transverse and conjugate axes , eccentricity , coordinates of the foci ; equation of the directrices & length of latus – rectum

1. $16x^2 - 9y^2 = 144$

ans : 6 , 8 , 5/3 , (± 5,0) , x = ± 9/5 ; 32/3

2. $9x^2 - 16y^2 = 144$

ans : 8 , 6 , 5/4 , (± 5,0) , x = ± 16/5 ; 9/2

3. $9x^2 - 25y^2 = 225$

ans :

10 , 6 , $\sqrt{34}/5$, $(\pm\sqrt{34}, 0)$, x = ± $25/\sqrt{34}$, 18/5

Q1. find equation of hyperbola referred to principal axes

1. length of the transverse axis & length of latus rectum are 18 & 14 respectively

Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

length of transverse axis = 18

$2a = 18$

$a = 9$

length of latus rectum = 14

$$\frac{2b^2}{a} = 14$$

$$\frac{b^2}{a} = 7$$

subs $a = 9$

$$\frac{b^2}{9} = 7$$

$b^2 = 63$

$\therefore$ the equation of the hyperbola is

$$\frac{x^2}{81} - \frac{y^2}{63} = 1$$

2. eccentricity = 3/2 & distance between foci = 6

Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

distance between foci = 6

$2ae = 6$

$ae = 3$

subs $e = 3/2$

$$a\frac{3}{2} = 3$$

$a = 2$

Now ; $b^2 = a^2(e^2 - 1)$

$$b^2 = 4\left(\frac{9}{4} - 1\right)$$

$$b^2 = 4x\ \frac{5}{4}$$

$b^2 = 5$

$\therefore$ the equation of the hyperbola is

$$\frac{x^2}{4} - \frac{y^2}{5} = 1$$

3. eccentricity = 3/2 & distance between foci = 12

Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

distance between foci = 6

$2ae = 12$

$ae = 6$

subs $e = 3/2$

$$a\frac{3}{2} = 6$$

$a = 4$

Now ; $b^2 = a^2(e^2 - 1)$

$$b^2 = 16\left(\frac{9}{4} - 1\right)$$

$$b^2 = 16\ x\ \frac{5}{4}$$

$b^2 = 20$

$\therefore$ the equation of the hyperbola is

$$\frac{x^2}{16} - \frac{y^2}{20} = 1$$

4. eccentricity = $\sqrt{2}$ & distance between foci = $8\sqrt{2}$

Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

distance between foci = $8\sqrt{2}$

$$2ae = 8\sqrt{2}$$

$$ae = 4\sqrt{2}$$

subs $e = \sqrt{2}$ $\quad a\sqrt{2} = 4\sqrt{2}$

$$a = 4$$

Now ; $b^2 = a^2(e^2 - 1)$

$b^2 = 16\,(2 - 1)$

$b^2 = 16$

$\therefore$ the equation of the hyperbola is

$$\frac{x^2}{16} - \frac{y^2}{16} = 1$$

5. eccentricity = 5/3 & distance between directrices = 18/5

Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

distance between directrices = 18/5

$$\frac{2a}{e} = \frac{18}{5}$$

$$\frac{a}{e} = \frac{9}{5}$$

subs $e = 5/3$ $\quad \frac{a}{5/3} = \frac{9}{5}$

$$\frac{3a}{5} = \frac{9}{5}$$

$$a = 3$$

Now ; $b^2 = a^2(e^2 - 1)$

$b^2 = 9\left(\frac{25}{9} - 1\right)$

$b^2 = 9 \times \frac{16}{9}$

$b^2 = 16$

$\therefore$ the equation of the hyperbola is

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

6. foci = $(\pm 8,0)$ & vertex at $(6,0)$

Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

foci $\equiv (\pm 8,0) \equiv (\pm ae,0)$

$\therefore$ $ae = 8$

vertex at $(6,0) \equiv (a, 0)$

$\therefore$ $a = 6$

subs in $ae = 8$

$6e = 8 \quad \therefore e = 4/3$

Now ; $b^2 = a^2(e^2 - 1)$

$b^2 = 36\left(\frac{16}{9} - 1\right)$

$b^2 = 36 \times \frac{7}{9}$

$b^2 = 28$

$\therefore$ the equation of the hyperbola is

$$\frac{x^2}{36} - \frac{y^2}{28} = 1$$

7. eccentricity = $\sqrt{2}$ &

passing through $(-5,3)$

Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Since hyperbola passes through $(-5,3)$, it must satisfy the equation

$$\frac{25}{a^2} - \frac{9}{b^2} = 1 \quad \ldots\ldots \quad (1)$$

$e = \sqrt{2}$ ..... given

Now ; $b^2 = a^2(e^2 - 1)$

$b^2 = a^2(2 - 1)$

$b^2 = a^2$

subs in (1) $\frac{25}{a^2} - \frac{9}{a^2} = 1$

$$a^2 = 16$$

$$\therefore\ b^2 = 16$$

$\therefore$ the equation of the hyperbola is

$$\frac{x^2}{16} - \frac{y^2}{16} = 1$$

8. conjugate axis = 8 and distance between foci = 10

Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

conjugate axis = 8

$$2b = 8$$

$$b = 4$$

distance between foci = 10

$$2ae = 10$$

$$ae = 5$$

$$e = 5/a$$

Now ; $b^2 = a^2(e^2 - 1)$

$$16 = a^2\left(\frac{25}{a^2} - 1\right)$$

$$16 = 25 - a^2$$

$$a^2 = 9$$

$\therefore$ the equation of the hyperbola is

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

9. conjugate axis = 5 and distance between foci = 13

Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

conjugate axis = 5

$$2b = 5$$

$$b = {}^5/_2$$

distance between foci = 13

$$2ae = 13$$

$$ae = {}^{13}/_2$$

$$e = {}^{13}/_{2a}$$

Now ; $b^2 = a^2(e^2 - 1)$

$$\frac{25}{4} = a^2\left(\frac{169}{4a^2} - 1\right)$$

$$\frac{25}{4} = a^2\left(\frac{169 - 4a^2}{4a^2}\right)$$

$$25 = 169 - 4a^2$$

$$4a^2 = 144$$

$$a^2 = 36$$

$\therefore$ the equation of the hyperbola is

$$\frac{x^2}{36} - \frac{4y^2}{25} = 1$$

10. transverse axis = $2\sqrt{2}$ and passing thro' (2,–3)

Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

transverse axis = $2\sqrt{2}$

$$2a = 2\sqrt{2}$$

$$a = \sqrt{2}$$

Since hyperbola passes through (2,–3) , it must satisfy the equation

$$\frac{4}{a^2} - \frac{9}{b^2} = 1 \quad \ldots\ldots \quad (1)$$

subs $a = \sqrt{2}$

$$\frac{4}{2} - \frac{9}{b^2} = 1 \quad \ldots\ldots \quad (1)$$

$$2 - 1 = \frac{9}{b^2}$$

$$b^2 = 9$$

$\therefore$ the equation of the hyperbola is

$$\frac{x^2}{2} - \frac{y^2}{9} = 1$$

11. transverse axis = 7 and passing thro' (5,–2)

Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

transverse axis = 7

$2a = 7$

$a = {}^7/_2$

Since hyperbola passes through (5,–2) , it must satisfy the equation

$$\frac{25}{a^2} - \frac{4}{b^2} = 1 \quad \ldots\ldots \quad (1)$$

subs $a = {}^7/_2$

$$\frac{25}{\frac{49}{4}} - \frac{4}{b^2} = 1 \quad \ldots\ldots \quad (1)$$

$$\frac{100}{49} - 1 = \frac{4}{b^2}$$

$$\frac{51}{49} = \frac{4}{b^2}$$

$$b^2 = \frac{196}{51}$$

∴ the equation of the hyperbola is

$$\frac{4x^2}{49} - \frac{51y^2}{196} = 1$$

12. length of latus rectum is equal to square of the length of conjugate axis , passes through (–2,3)

Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

latus rectum = square of conjugate axis

$$\frac{2b^2}{a} = (2b)^2$$

$$\frac{2b^2}{a} = 4b^2$$

$2 = 4a \qquad a = 1/2$

Since hyperbola passes through (–2,3) , it must satisfy the equation

$$\frac{4}{a^2} - \frac{9}{b^2} = 1 \quad \ldots\ldots \quad (1)$$

subs a = 1/2

$$\frac{4}{\frac{1}{4}} - \frac{9}{b^2} = 1 \quad \ldots\ldots \quad (1)$$

$$16 - 1 = \frac{9}{b^2}$$

$$b^2 = \frac{9}{15} = \frac{3}{5}$$

∴ the equation of the hyperbola is

$$4x^2 - \frac{5y^2}{3} = 1$$

13. passing through (2,1) & (4,3)

Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Since hyperbola passes through (2,1) & (4,3), it must satisfy the equation

$$\frac{4}{a^2} - \frac{1}{b^2} = 1 \quad \ldots\ldots \quad (1)$$

$$\frac{16}{a^2} - \frac{9}{b^2} = 1 \quad \ldots\ldots \quad (2)$$

(1) x 4 $\quad \frac{16}{a^2} - \frac{4}{b^2} = 4 \quad \ldots\ldots \quad (1)$

$$\frac{16}{a^2} - \frac{9}{b^2} = 1 \quad \ldots\ldots \quad (2)$$

$$\frac{5}{b^2} = 3$$

$$b^2 = 5/3$$

subs in 1

$$\frac{4}{a^2} - \frac{1}{\frac{5}{3}} = 1$$

$$\frac{4}{a^2} - \frac{3}{5} = 1$$

$$\frac{4}{a^2} = 1 + \frac{3}{5}$$

$$\frac{4}{a^2} = \frac{8}{5} \qquad a^2 = \frac{5}{2}$$

∴ the equation of the hyperbola is

$$\frac{2x^2}{5} - \frac{3y^2}{5} = 1$$

14. passing through (6, −1) & (−8,2√2)

Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Since hyperbola passes through (6, −1) & (−8,2√2), it must satisfy the equation

$$\frac{36}{a^2} - \frac{1}{b^2} = 1 \quad \ldots\ldots \quad (1)$$

$$\frac{64}{a^2} - \frac{8}{b^2} = 1 \quad \ldots\ldots \quad (2)$$

(1) x 8 $\frac{288}{a^2} - \frac{8}{b^2} = 8 \quad \ldots\ldots \quad (1)$

$$\frac{64}{a^2} - \frac{8}{b^2} = 1 \quad \ldots\ldots \quad (2)$$

$$\frac{224}{a^2} = 7$$

$$a^2 = 32$$

subs in (2)

$$\frac{64}{32} - \frac{8}{b^2} = 1 \quad \ldots\ldots \quad (2)$$

$$2 - 1 = \frac{8}{b^2}$$

$$b^2 = 8$$

∴ the equation of the hyperbola is

$$\frac{x^2}{32} - \frac{y^2}{8} = 1$$

15.foci of hyperbola coincides with the foci of ellipse $9x^2 + 16y^2 = 144$; $e = \sqrt{2}$

ELLIPSE : $9x^2 + 16y^2 = 144$

$$\frac{9x^2}{144} + \frac{16y^2}{144} = 1$$

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

$$a^2 = 16 \text{ , } b^2 = 9$$

$$b^2 = a^2(1 - e^2)$$

$$9 = 16(1 - e^2)$$

$$\frac{9}{16} = 1 - e^2$$

$$e^2 = 1 - \frac{9}{16}$$

$$e^2 = \frac{7}{16} \qquad e = \frac{\sqrt{7}}{4}$$

$$\text{foci} \equiv (\pm ae \text{ , } 0) = (\pm \sqrt{7},0)$$

HYPERBOLA

Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Since foci of hyperbola coincides with that of ellipse

$$\text{foci} \equiv (\pm ae \text{ , } 0) = (\pm \sqrt{7},0)$$

$$ae = \sqrt{7}$$

$e = \sqrt{2}$ …….. given

$$\therefore \quad a = \sqrt{7}/\sqrt{2}$$

Now ; $b^2 = a^2(e^2 - 1)$

$$b^2 = \frac{7}{2}(2 - 1)$$

$$b^2 = \frac{7}{2}$$

∴ the equation of the hyperbola is

$$\frac{2x^2}{7} - \frac{2y^2}{7} = 1$$

Q2.

01. for : $\frac{x^2}{100} - \frac{y^2}{25} = 1$

Prove that :

a) $e = \sqrt{5}/2$ b) SA.S'A = 25 where S, S' are foci , A is vertex

$\frac{x^2}{100} - \frac{y^2}{25} = 1$

$a^2 = 100 \quad ; \quad a = 10$

$b^2 = 25 \quad ; \quad b = 5$

$b^2 = a^2(e^2 - 1)$

$25 = 100(e^2 - 1)$

$\frac{1}{4} = e^2 - 1$

$e^2 = \frac{5}{4} \qquad e = \frac{\sqrt{5}}{2}$

$ae = 10 \times \frac{\sqrt{5}}{2} = 5\sqrt{5}$

$S \equiv (ae,0) \equiv (5\sqrt{5},0)$

$S' \equiv (-ae,0) \equiv (-5\sqrt{5},0)$

$A \equiv (a,0) \equiv (10,0)$

$SA = \sqrt{(5\sqrt{5} - 10)^2 + (0 - 0)^2}$

$= 5\sqrt{5} - 10$

$S'A = \sqrt{(-5\sqrt{5} - 10)^2 + (0 - 0)^2}$

$= \sqrt{(5\sqrt{5} + 10)^2}$

$= 5\sqrt{5} + 10$

$SA . S'A = (5\sqrt{5} - 10)(5\sqrt{5} + 10)$

$= 125 - 100$

$= 25$ ..... Proved

02. for hyperbola : $x^2 - 2y^2 = 1$, find

a) e b) |SA – S'A| c) SA.S'A

where S and S' are the foci and A is the vertex

$x^2 - 2y^2 = 1$

$x^2 - \frac{y^2}{\frac{1}{2}} = 1$

$a^2 = 1 \quad ; \quad a = 1$

$b^2 = {}^1/_2 \quad ; \quad b = {}^1/\sqrt{2}$

$b^2 = a^2(e^2 - 1)$

$\frac{1}{2} = e^2 - 1$

$\frac{3}{2} = e^2 \qquad e = \frac{\sqrt{3}}{\sqrt{2}}$

$ae = 1 \times \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}}$

$S \equiv (ae,0) \equiv ({}^{\sqrt{3}}/\sqrt{2},0)$

$S' \equiv (-ae,0) \equiv (-{}^{\sqrt{3}}/\sqrt{2},0)$

$A \equiv (a,0) \equiv (1,0)$

$SA = \sqrt{({}^{\sqrt{3}}/\sqrt{2} - 1)^2 + (0 - 0)^2}$

$= {}^{\sqrt{3}}/\sqrt{2} - 1$

$S'A = \sqrt{(-{}^{\sqrt{3}}/\sqrt{2} - 1)^2 + (0 - 0)^2}$

$= \sqrt{({}^{\sqrt{3}}/\sqrt{2} + 1)^2}$

$= {}^{\sqrt{3}}/\sqrt{2} + 1$

$|SA - S'A|$

$= |({}^{\sqrt{3}}/\sqrt{2} - 1) - ({}^{\sqrt{3}}/\sqrt{2} + 1)|$

$= |{}^{\sqrt{3}}/\sqrt{2} - 1 - {}^{\sqrt{3}}/\sqrt{2} - 1|$

$= 2$

$SA . S'A = (\sqrt{3}/\sqrt{2} + 1)(\sqrt{3}/\sqrt{2} + 1)$

$= {}^3/2 - 1$

$= {}^1/2$

03. if e_1 and e_2 are the eccentricities of hyperbolas

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ & $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$

then prove that : $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$b^2 = a^2(e_1^2 - 1)$

$\frac{b^2}{a^2} = e_1^2 - 1$

$\frac{b^2}{a^2} + 1 = e_1^2$

$e_1^2 = \frac{a^2 + b^2}{a^2}$

$\frac{1}{e_1^2} = \frac{b^2}{a^2 + b^2}$

$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$

$a^2 = b^2(e_2^2 - 1)$

$\frac{a^2}{b^2} = e_2^2 - 1$

$\frac{a^2}{b^2} + 1 = e_2^2$

$e_2^2 = \frac{a^2 + b^2}{a^2}$

$\frac{1}{e_2^2} = \frac{a^2}{a^2 + b^2}$

HENCE

$\frac{1}{e_1^2} + \frac{1}{e_2^2} = \frac{b^2}{a^2 + b^2} + \frac{a^2}{a^2 + b^2} = 1$

Q3.

find length of transverse and conjugate axes , eccentricity , coordinates of the foci ; equation of the directrices & length of latus – rectum

1. $16x^2 - 9y^2 = 144$

$\frac{16x^2}{144} - \frac{9y^2}{144} = 1$

$\frac{x^2}{9} - \frac{y^2}{16} = 1$

$a^2 = 9 \quad a = 3$

$b^2 = 16 \quad b = 4$

$b^2 = a^2(e^2 - 1)$

$16 = 9(e^2 - 1)$

$\frac{16}{9} = e^2 - 1$

$\frac{16}{9} + 1 = e^2$

$e^2 = \frac{25}{9} \qquad e = \frac{5}{3}$

$ae = 3 \times \frac{5}{3} = 5$

$\frac{a}{e} = \frac{3}{\frac{5}{3}} = \frac{9}{5}$

length of transverse axis = $2a$ = 6

length of conjugate axis = $2b$ = 8

foci ≡ $(\pm ae , 0)$ ≡ $(\pm 5 , 0)$

eq. of directrices : $x = \pm \frac{a}{e}$

$x = \pm \frac{9}{5}$

length of latus rectum = $\frac{2b^2}{a} = \frac{32}{3}$

2. $9x^2 - 16y^2 = 144$

$$\frac{9x^2}{144} - \frac{16y^2}{144} = 1$$

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

$a^2 = 16 \quad a = 4$

$b^2 = 9 \quad b = 3$

$b^2 = a^2(e^2 - 1)$

$9 = 16(e^2 - 1)$

$\frac{9}{16} = e^2 - 1$

$\frac{9}{16} + 1 = e^2$

$e^2 = \frac{25}{16} \qquad e = \frac{5}{4}$

$ae = 4 \times \frac{5}{4} = 5$

$\frac{a}{e} = \frac{4}{\frac{5}{4}} = \frac{16}{5}$

length of transverse axis = $2a$ = 8

length of conjugate axis = $2b$ = 6

foci ≡ $(\pm ae\ ,\ 0)$ ≡ $(\pm 5\ ,\ 0)$

eq. of directrices : $x = \pm \frac{a}{e}$

$x = \pm \frac{16}{5}$

length of latus rectum = $\frac{2b^2}{a} = \frac{9}{2}$

3. $9x^2 - 25y^2 = 225$

$$\frac{9x^2}{225} - \frac{25y^2}{225} = 1$$

$$\frac{x^2}{25} - \frac{y^2}{9} = 1$$

$a^2 = 25 \quad a = 5$

$b^2 = 9 \quad b = 3$

$b^2 = a^2(e^2 - 1)$

$9 = 25(e^2 - 1)$

$\frac{9}{25} = e^2 - 1$

$\frac{9}{25} + 1 = e^2$

$e^2 = \frac{34}{25} \qquad e = \frac{\sqrt{34}}{5}$

$ae = 5 \times \frac{\sqrt{34}}{5} = \sqrt{34}$

$\frac{a}{e} = \frac{5}{\frac{\sqrt{34}}{5}} = \frac{25}{\sqrt{34}}$

length of transverse axis = $2a$ = 10

length of conjugate axis = $2b$ = 6

foci ≡ $(\pm ae\ ,\ 0)$ ≡ $(\pm\sqrt{34}\ ,\ 0)$

eq. of directrices : $x = \pm \frac{a}{e}$

$x = \pm \frac{25}{\sqrt{34}}$

length of latus rectum = $\frac{2b^2}{a} = \frac{18}{5}$